1 Production Process Models

Example 1.1. DoggyChow
DoggyChow (DC) produces 3 premium grades of dog food they’ve internally labeled A, B and C. Each day, they can sell up to 3000 pounds of each product at the following prices per pound:
product A – $10
product B – $12
product C – $20
DC purchases raw organic, free-range beef at $5/lb. Each pound of beef can be used to produce either 1 lb. of A or 1 lb. of B. For a cost of $3/lb., product A can be converted to .6 lb. of product B and .4 lb. of product C. For a cost of $2/lb., product B can be converted to .8 lb. of product C. Formulate an LP to maximize profit.

Step 1: Fix our decision variables.
$P_A =$ (number of pounds of beef purchased to make A)
$P_B =$ (number of pounds of beef purchased to make B)
$C_{AB} =$ (number of pounds of A converted to B)
$C_{AC} =$ (number of pounds of A converted to C)
$C_{BC} =$ (number of pounds of B converted to C)
$S_A =$ (number of pounds of A sold)
$S_B =$ (number of pounds of B sold)
$S_C =$ (number of pounds of C sold)
**Step 2:** Create the objective function.

\[ \text{Profit} = 10S_A + 12S_B + 20S_C - \left[ 5(P_A + P_B) + 3(C_{AB} + C_{AC}) + 2C_{BC} \right] \]

**Step 3:** Write down all the constraints.
The easy ones are \( S_A \leq 3000, \ S_B \leq 3000, \) and \( S_C \leq 3000 \) and nonnegativity.

Now we must encode the relationships between the sold amount and the amount that is used for conversions.

For type \( A \) we have
\[ S_A = P_A - C_{AB} - C_{AC}. \]

For type \( B \) we have
\[ S_B = P_B - C_{BC} + .6(C_{AB} + C_{AC}). \]

For type \( C \) we have
\[ S_C = .4(C_{AC} + C_{AB}) + .8C_{BC}. \]