SM212, Prof Joyner, 2-27-2004
Practice Test 2

PROBLEM 1: Find the general form of $y_p$. Do not solve for the undetermined coefficients. Do not solve the ODE.

(a) $y'' - 2y' + y = xe^x$,  
(b) $y'' - 3y' + 2y = e^x + e^{2x} + \sin(x)$.  

(Ans: (a) $y_p = A_1x^3e^x + A_2xe^x + A_3xe^x$, (b) $y_p = A_1xe^x + A_2xe^{2x} + A_3\cos(x) + A_4\sin(x)$.)

PROBLEM 2: An RLC circuit has a 1 henry inductor in series with a 4 ohm resistor and a $1/5$ farad capacitor. A battery of $\sin(t)$ volts is attached.

(a) Assume that the initial charge and curent is zero. Find the current at time $t$.

(b) Identify the transient and steady state terms.

(Ans: (a) $q(t) = \frac{1}{8}\exp(-2t)\cos(t) + \frac{1}{8}\exp(-2t)\sin(t) - \frac{1}{8}\cos(t) + \frac{1}{8}\sin(t)$, 
$i(t) = -\frac{1}{8}\exp(-2t)\cos(t) - \frac{3}{8}\exp(-2t)\sin(t) + \frac{1}{8}\cos(t) + \frac{3}{8}\sin(t)$, (b) transient: $\frac{1}{8}\exp(-2t)\cos(t) + \frac{1}{8}\exp(-2t)\sin(t)$, steady state: $-\frac{1}{8}\cos(t) + \frac{1}{8}\sin(t)$)

PROBLEM 3: Solve  

$x'' + 4x = \tan(2t)$, $x(0) = 0$, $x'(0) = 0$.

(Ans: $x(t) = \frac{1}{4}\sin(2t) - \frac{1}{4}\cos(2t)\ln\left(\frac{1+\sin(2t)}{\cos(2t)}\right)$)
PROBLEM 4: A 1 kg object is attached to a spring suspended from the ceiling. The spring has spring constant 4 and has an external force of \( \sin(\gamma t) \) acting on it.

(a) Find the \( \gamma \) for which the displacement is in a state of pure resonance (i.e., has the same period as the oscillations of the homogeneous part of the solution).

(b) For this \( \gamma \) in (a), find the displacement if the mass is initially released with a 1 m/s downward velocity at equilibrium. (If you don’t know the answer to (a), just take \( \gamma = 1 \)).

\[(\text{Ans: } (a) \; \gamma = 2, \; (b) \; x(t) = -\frac{1}{4} t \cos(2t) + \frac{5}{8} \sin(2t))\]

PROBLEM 5: Solve \( y'''' - y = e^x \).

\[(\text{Ans: } y(x) = \frac{1}{4} x \exp(x) - \frac{3}{8} \exp(x) + c_1 \exp(x) + c_2 \sin(x) + c_3 \cos(x) + c_4 \exp(-x))\]

PROBLEM 6: An RLC circuit has a 1/2 henry inductor in series with a 3 ohm resistor and a 1/5 farad capacitor. A battery of 39\( \sin(t) \) volts is attached.

1. Assume that the initial charge and current is zero. Find the charge at time \( t \).

2. Identify the transient and steady state terms.

\[(\text{Ans: } 6e^{-3t} \sin(t) + 4e^{-3t} \cos(t) - 4 \cos(t) + 6 \sin(t), \; \text{transient}=\; 6e^{-3t} \sin(t) + 4e^{-3t} \cos(t), \; \text{steady state}=\; -4 \cos(t) + 6 \sin(t))\]