Name:

Closed book, closed notes, TI92 okay. No discussion of this exam until after the end of 6th period.

1. Using variation of parameters, solve

\[ x' = -4y + 1, \]
\[ y' = -x - 3. \]

2. Let \( f(x) \) be the function defined on \( 0 \leq x \leq 2 \) by \( f(x) = 1 \), for \( 0 \leq x < 1 \), \( f(x) = -1 \), for \( 1 \leq x \leq 2 \).
   a) Find the sine series of \( f(x) \), \( L = 2 \),
   b) Write down the sum of the first 3 non-zero terms,
   c) Graph the function to which this series converges for \( -2 < x < 2 \).
3. Using separation of variables, solve

\[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \]

\[ u(x, 0) = -\sin\left(\frac{\pi x}{3}\right) + 2\sin(\pi x), \]

\[ u(0, t) = u(3, t) = 0. \]

You need not show the steps of the separation of variables process.

4. Using separation of variables, find all product solutions to

\[ \frac{1}{2} \frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}. \]

BONUS: Let \( f(x) \) be as in \# 2. Solve

\[ \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \]

\[ u(x, 0) = f(x), \]

\[ u(0, t) = u(2, t) = 0. \]