SM212P Test 4, 4-25-2003
Prof Joyner Name:

No math tables, no TI92s, closed books, closed notes. No discussion of this exam until after 2nd period. Work individually.

1. Solve

\[
\begin{align*}
\frac{\partial^2 u}{\partial x^2} &= \frac{\partial^2 u}{\partial t^2}, \\
u(x, 0) &= \sin(\pi x) - 5\sin(3\pi x), \quad 0 < x < 3, \\
u_t(x, 0) &= 0, \quad 0 < x < 3, \\
u(0, t) &= u(3, t) = 0.
\end{align*}
\]

You do not need to show all steps of the separation of variables process for this problem.
2. Let

\[ f(x) = \begin{cases} 
0, & 0 < x < \pi/2, \\
1, & \pi/2 < x < \pi. 
\end{cases} \]

(a) Find the sine series of \( f(x) \).
(b) Find its value at \( x = 0, \pi/2, 101\pi \).

3. (a) Find the Fourier series of

\[ f(x) = \begin{cases} 
1, & -3 < x < 0, \\
2, & 0 < x < 3/2 \\
3, & 3/2 < x < 3, 
\end{cases} \]

extended to the real line with period 6, \( f(x) = f(x + 6) \).

(b) Find the value of the function to which the Fourier series in (a) converges to at \( x = -6, 0, 1, 9 \).

(c) Find the sum of the first 4 non-zero terms in the Fourier series in (a).
4. (a) Using separation of variables, solve
\[
\begin{aligned}
\frac{1}{\pi^2} \frac{\partial^2 u}{\partial x^2} &= \frac{\partial u}{\partial t}, \\
u(x, 0) &= 3 \cos(\pi x), \quad 0 < x < 3, \\
u_x(0, t) &= u_x(3, t) = 0.
\end{aligned}
\]
Show all steps.
(b) Find the temperature of the wire at the center of the wire after 1 second.