Micromechanical Analysis of a Lamina
Predictions of Strength and Elastic Constants for UD Lamina

A variety of methods have been used to predict properties of composite materials.

The approaches used fall into the following general categories:

- Mechanics of materials
- Numerical
- Self-consistent field
- Bounding (variational approach)
- Semi-empirical
- Experiential
Predictions of Strength and Elastic Constants for UD Lamina

- Mechanics of materials approach is based on simplifying assumptions of either uniform strain or uniform stress in the constituents.
Predictions of Strength and Elastic Constants for UD Lamina

Numerical approaches using finite differences, finite element, or boundary element methods yield the best predictions, however they are time consuming and they do not yield closed form expressions (families of curves.)
Predictions of Strength and Elastic Constants for UD Lamina

- In the self-consistent field approach a simplified composite model is considered consisting of a typical fiber surrounded by a cylindrical matrix phase.
- This composite element is considered embedded in an infinite, homogeneous medium whose properties are identical to the average properties of the composite material.
- Classical elasticity theory has been used to obtain closed form solutions for the various elastic constants of the composite.
- Because of gross geometric simplifications involved, this approach tends to underestimate composite properties for high $V_f$.
Predictions of Strength and Elastic Constants for UD Lamina

Variational methods based on energy principles have been developed to establish bounds on effective properties.
Predictions of Strength and Elastic Constants for UD Lamina

- Semi-empirical relationships have been developed to circumvent the difficulties with the theoretical approaches and to facilitate computation.
- So-called Halpin-Tsai relationships have a consistent form for all properties and represent an attempt at judicious interpolation between the series and parallel models used in the mechanics of materials approach or between the upper and lower bounds of the variational approach.

© 2003, P. Joyce
# Resin Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3501-6</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>1.2 g/cc</td>
</tr>
<tr>
<td>Tensile Modulus</td>
<td>3.4 GPa</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>59 MPa</td>
</tr>
<tr>
<td>% Elong.</td>
<td>3.3</td>
</tr>
</tbody>
</table>
# Fiber Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM6</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>1.80</td>
</tr>
<tr>
<td>Tensile Modulus</td>
<td>290 GPa</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>4480 MPa</td>
</tr>
<tr>
<td>% Elong.</td>
<td>0.2%</td>
</tr>
</tbody>
</table>
Role of Fiber Reinforcement
(Review)

- The mechanical properties of fiber reinforced PMCs dominated by the contribution of the fiber to the composite
- The **four main factors** that govern the fiber’s contribution are:
  - The basic mechanical properties of the fiber itself
  - The orientation of the fiber in the composite
  - The amount of fiber in the composite
  - The surface interaction of the fiber and resin
Micromechanics of UD Lamina—Elastic Behavior

- Examine how we can predict the elastic behavior of a unidirectional lamina using simple micromechanics.
Micromechanics – Elastic Behavior

\[ F_c = F_m + F_f \]
Using the definition of stress, \( \bar{F} = \sigma A \)

Substituting, \( \sigma_c A_c = \sigma_m A_m + \sigma_f A_f \)

Rearranging, \( \sigma_c = \sigma_m \frac{A_m}{A_c} + \sigma_f \frac{A_f}{A_c} \)

where, \( \frac{A_m}{A_c} \) and \( \frac{A_f}{A_c} \) are the area fractions of the matrix and fiber phases, respectively.
Micromechanics – Elastic Behavior

If the composite, matrix, and fiber phase lengths are all equal, then

\[ \frac{A_m}{A_c} \]

is equivalent to the volume fraction of the matrix, \( V_m \); and likewise for the fiber volume fraction, \( V_f = \frac{A_f}{A_c} \)

\[ \sigma_c = \sigma_m V_m + \sigma_f V_f \]
Micromechanics – Elastic Behavior

- Assuming a perfect fiber/matrix bond, then isostrain state $\varepsilon_c = \varepsilon_m = \varepsilon_f$

- Then using Hooke’s Law, $\sigma = E \varepsilon$

$$E_c = E_m V_m + E_f V_f$$

Rule of Mixtures

© 2003, P. Joyce
Example

- $E_m = 3.4$ GPa
- $E_f = 290$ GPa
- $V_f = 60\%$
- $E_c = 175$ GPa (25 Msi)
- Check against literature. . .
Micromechanics – Elastic Behavior

What about transverse loading?
Micromechanics – Elastic Behavior

- Use Statics, \( F_c = F_m = F_f \)
- And \( A_c = A_m = A_f \)
- Therefore \(- \sigma_c = \sigma_m = \sigma_f = \sigma \)

- However, \( \varepsilon_c = \varepsilon_m V_m + \varepsilon_f V_f \)
- Using Hooke’s Law again, and substituting,

\[
\frac{1}{E_c} = \frac{V_m}{E_m} + \frac{V_f}{E_f}
\]
Micromechanics – Elastic Behavior

Which reduces to,

\[ E_c = \frac{E_m E_f}{V_m E_f + V_f E_m} = \frac{E_m E_f}{(1-V_f)E_f + V_f E_m} \]

The matrix modulus is usually replaced by

\[ E'_m = \frac{E_m}{1-\nu_m^2} \]

to account for the constraint imposed on the matrix by the fibers in the fiber direction.
Micromechanics – Elastic Behavior

\[ E_c = \frac{E_f E_m'}{V_f E_m' + V_m E_f} \]

- The mechanics of materials prediction above tends to underestimate the transverse modulus.
- Halpin-Tsai semi-empirical relationship is a practical alternative.
Micromechanics – Elastic Behavior

- The behavior of UD composites under in-plane (longitudinal) shear loading is also dominated by the matrix properties and the local stress distributions.
- The mechanics of materials approach uses a series model under uniform stress and yields the following relation:

\[
\frac{1}{G_{12}} = \frac{V_f}{G_{12_f}} + \frac{V_m}{G_m} \quad \text{or} \quad G_{12} = \frac{G_{12_f} G_m}{V_f G_m + V_m G_{12_f}}
\]

As in the case of transverse modulus, this approach tends to underestimate the in-plane shear modulus, use instead the Halpin-Tsai semi-empirical relation.
Example

- $E_m = 3.4$ GPa
- $E_{f1} = 290$ GPa
- $E_{f2} = 14$ GPa
- $V_f = 60\%$
- $E_{c1} = 175.4$ GPa
- $E_{c2} = 6.23$ GPa
- Check against literature...
Micromechanics – Elastic Behavior

The rule of mixtures prediction for the major (longitudinal) Poisson’s ratio is also very close to all other predictions and experimental results.

\[ \nu_{12} = V_f \nu_{12f} + V_m \nu_m \]
Micromechanics – Strength of UD Lamina

- The failure mechanisms and processes on a micromechanical scale vary with type of loading and are intimately related to the properties of the constituents (i.e. fiber, matrix, and interface-interphase.)
Micromechanics – Strength

- Under longitudinal tension, the phase with the lower ultimate strain will fail first.
- For perfectly bonded fibers, the average longitudinal stress in the composite, $\sigma_1$, is given by the rule of mixtures as

$$\sigma_1 = \sigma_f V_f + \sigma_m V_m$$
Micromechanics – Strength

Under the simple deterministic assumption of uniform strengths, two cases are distinguished depending on the relative magnitudes of the ultimate tensile strains of the constituents.

When the ultimate tensile strain of the fiber is lower than that of the matrix, the composite will fail when its longitudinal strain reaches the ultimate tensile strain of the fiber.

\[ \sigma_{c_t} \approx \sigma_{f_t} V_f + \sigma_{m} V_m \]

Case of Fiber Dominated Strength
Assuming LE behavior for the constituents,

\[
\sigma_{c_t} \equiv \sigma_{f_t} V_f + E_m \varepsilon_{f t}^u V_m
\]

\[
\sigma_{c_t} = \sigma_{f_t} \left( V_f + V_m \frac{E_m}{E_f} \right)
\]

Assuming \( E_f \gg E_m \) and \( V_f \) is reasonable.

\[
\sigma_{c_t} = \sigma_{f_t} V_f
\]
Micromechanics – Strength

Alternatively,

When the ultimate tensile strain of the matrix is lower than that of the fiber, the composite will fail when its longitudinal strain reaches the ultimate tensile strain of the matrix.

\[
\sigma_{ci} \approx \sigma_f V_f + \sigma_m V_m
\]

\[
\sigma_{ci} \approx \sigma_m \left( V_f \frac{E_f}{E_m} + V_m \right)
\]

Case of Matrix Dominated Strength
Micromechanics – Strength

- These results do not take into consideration the statistical distribution of fiber and matrix strengths.
- In the case of fiber dominated strength, for example, fiber strength varies from point to point and from fiber to fiber.
- Not all fibers fail simultaneously...
- Initial fiber breaks induce nonuniform stress state...
Micromechanics – Strength

- Compressive failure is associated with microbuckling or kinking of the fibers within the matrix.
- Many complicated formulations in the literature. . .
The most critical loading of a UD composite is transverse tensile loading.

This type of loading results in high stress and strain concentrations in the matrix and interface/interphase.

Stress distributions around the fiber can be obtained analytically by finite element, finite difference, complex variable, or boundary element methods. . .

The critical stresses and strains usu. occur at the fiber/matrix interface.

Characterized by stress concentrations factors. . ., also strain concentration factors.
References