# Lesson 10: Part 1

- Turbulence
  - – Understand the basic characteristics of turbulence
  - – Understand and be able to apply the methods to describe it
    - Reynolds Decomposition
    - Reynolds Averaging

# Turbulence is a challenging and still very open area of study!

- In 1932, Horace Lamb, a British physicist said:
  - *"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics and the other is the turbulent motion of fluids. And about the former I am really rather optimistic."*

- *"Quite consistently, it has been said that turbulence is the invention of the Devil, put on Earth to torment us."*

# Sources of Turbulence

- Convection
  - An unstable environment will circulate vertically until it is stable
- Mechanical
  - Wind shear
  - Large gradients appear at the boundary due to the no-slip boundary condition

# The Nature of Turbulence

- Consider turbulence as a cascade of energy
  - Large eddies pass KE to smaller and smaller eddies where it is eventually dissipated into heat
- The energy is transferred by stretching and rotation of the eddies
  - There is continuous generation of turbulence to balance the dissipation

# The Nature of Turbulence

- Difficult to give a precise definition of turbulence; we can list some characteristics of turbulent flows:
  - Irregular (nonlinear)
  - Diffusive
  - Dissipative
  - Large Reynolds Number
  - 3D vorticity fluctuations

# The Nature of Turbulence

- Irregularity: randomness of all turbulent flows. Make a deterministic approach to turbulent problems impossible
  - **Statistical Methods**

- Diffusivity: causes rapid mixing in all turbulent flows
  - Increases momentum transfer between wind and ocean currents
  - The source of the resistance of flow in pipelines
  - Prevents boundary-layer separation on airfoils at large angles of attack

# The Nature of Turbulence

- Reynolds Number: denotes the relative importance of the viscous and inertial forces in the EOM

$$R_e = \frac{inertial}{viscous} = \frac{UL}{\nu}$$

# The Nature of Turbulence

- Dissipation: turbulent flows are always dissipative

Viscous Shear stress → deformation → KE to Internal energy

- Continuum: turbulence is a continuum phenomenon, governed by the equations of fluid mechanics. Even the smallest scales in a turbulent flow are far larger than any molecular length scale

# Turbulent Flows

- Turbulent flows are *flows*
- Turbulence is *not* a feature of fluids, but rather of *fluid* ***flows***
  - If $R_e$ is large enough, the dynamics of turbulence is the same in all fluids as the turbulent flows are not controlled by the molecular properties of the fluid
  - EOM are nonlinear, each flow has unique characteristics associated with the IC and BC
  - No general solution to the turbulent flow problem exists

# Analysis Methods

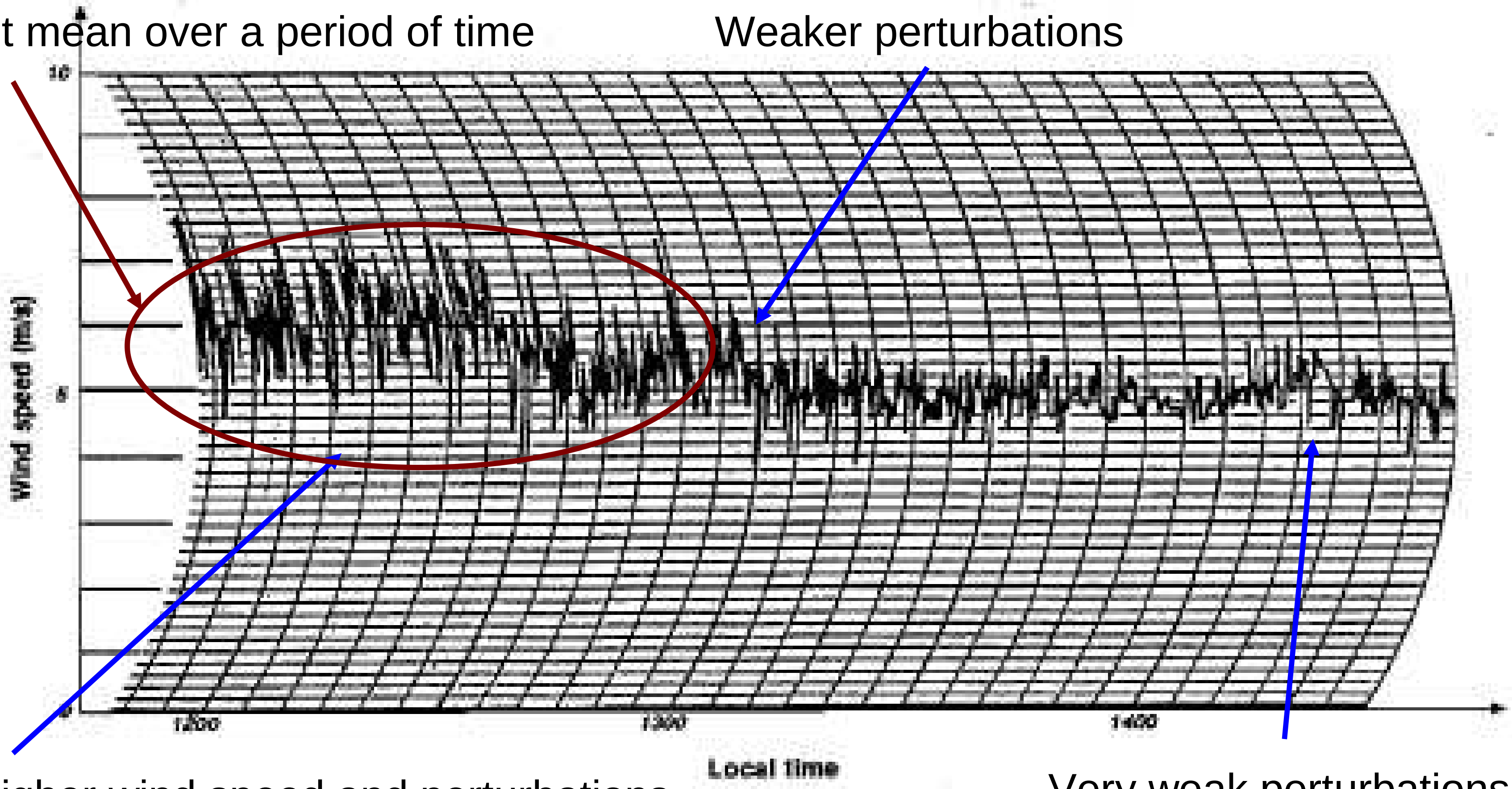
- Simplification through *scale analysis is not possible*
  - Turbulence is important in the boundary/surface layer so they *must* scale as large as the other remaining terms
  - Length scales of turbulence range from $10^{-3}$ to $10^3$ m

# Analysis Methods

- Stochastic Method: due to the **random nature** of turbulence, a deterministic solution to the EOM for turbulent flows is impossible. We must use a statistical method

- Dimensional Analysis: the most powerful tool in the study of turbulence. The arguments for similarity is based on the physical understanding of the problem. Experimental data is needed for a final solution.

# Signatures of Turbulence



Get mean over a period of time

Weaker perturbations

Higher wind speed and perturbations

Very weak perturbations

Fig. 3.1 Trace of wind speed observed in early afternoon. (from Stull 1988)

# Signatures of Turbulence

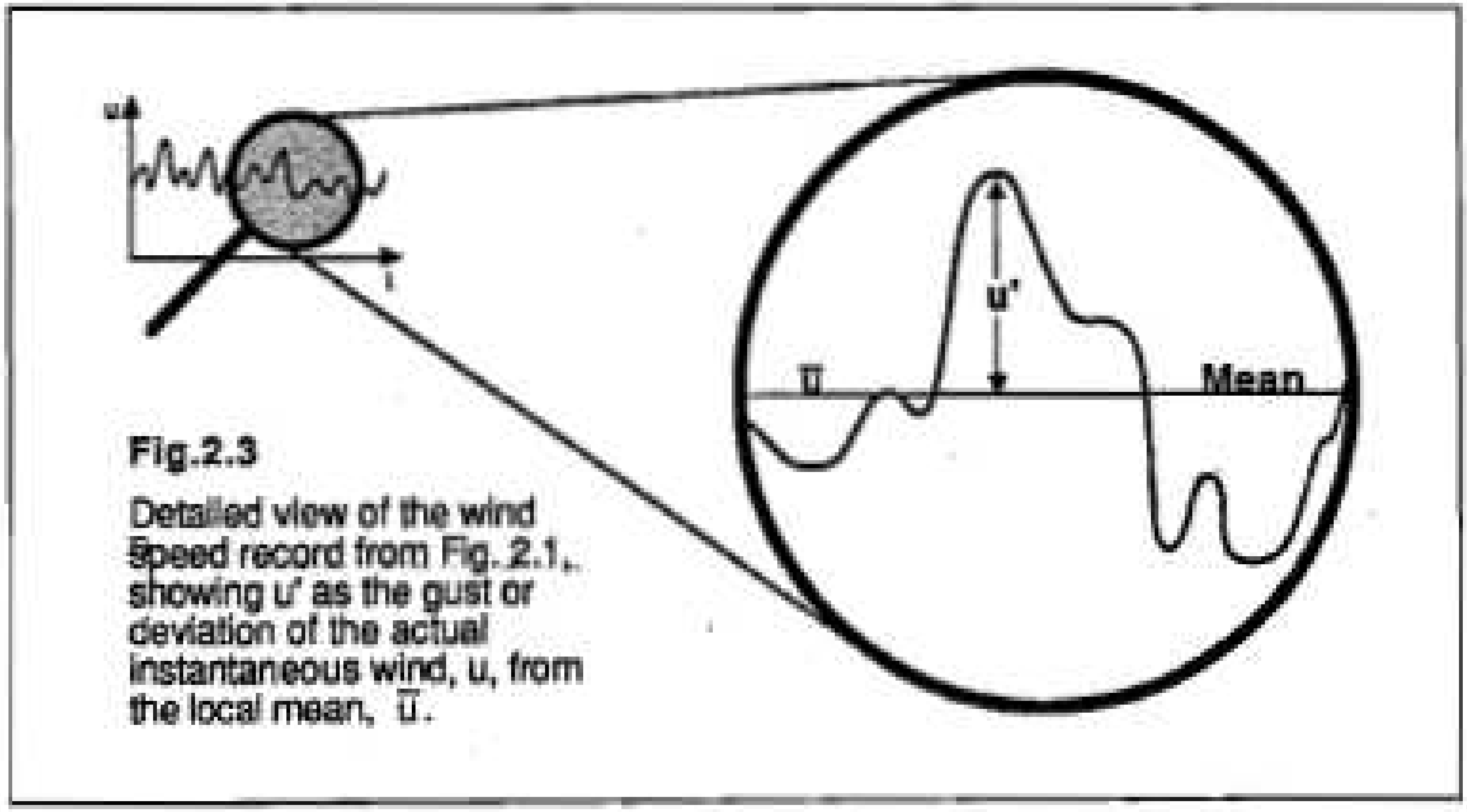


**Fig.2.3**

Detailed view of the wind speed record from Fig. 2.1, showing u' as the gust or deviation of the actual instantaneous wind, u, from the local mean, ū.

$$u' = u - \bar{u}$$

# Reynolds Decomposition

- One method to dealing with the complexity of turbulent flows is through an averaging technique - Reynolds Decomposition
- All variables in a flow can be decomposed into a basic state and turbulent part
  - Basic state ($\overline{c}$)
  - Turbulent ($c'$)

$$c = \overline{c} + c'$$

# Reynolds Decomposition

- Basic state ($\overline{c}$)
  - Slow variations (hours or longer)
  - Found by averaging the flow over a sufficiently long period (30 min or so)
    - Average out all the turbulent fluxes
    - Not so long as to average out any trends
- Turbulence ($c'$)
  - Short variations (seconds and minutes)
  - Average of these terms is zero

# Reynolds Decomposition

$$u = \overline{u} + u'$$

$$v = \overline{v} + v'$$

$$w = \overline{w} + w'$$

$$c = \overline{c} + c'$$

# Reynolds Averaging: Rules

- If A and B are two variables and c is a constant:

$$A = \overline{A} + a'$$

$$B = \overline{B} + b'$$

- Then:

$$c = \overline{c}$$

$$\overline{(cA)} = c\overline{A}$$

$$\overline{(\overline{A})} = \overline{A}$$

$$\overline{(\overline{A}B)} = \overline{A}\,\overline{B}$$

$$\overline{(A+B)} = \overline{A} + \overline{B}$$

$$\overline{\left(\frac{dA}{dt}\right)} = \frac{d\overline{A}}{dt}$$

$$\overline{a'} = 0, \quad \overline{a'b'} \neq 0$$

# Reynolds Averaging

- Begin with the x-component of EOM
- Plug in means and perturbations
  - i.e. Reynolds decomposition
- Average the equation
  - i.e. use Reynolds averaging

# Turbulent Equations of Motion (x-component)

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\,\frac{\partial \overline{u}}{\partial x} + \overline{v}\,\frac{\partial \overline{u}}{\partial y} + \overline{w}\,\frac{\partial \overline{u}}{\partial z} - f\overline{v} =$$

$$-\,\overline{a}\,\frac{\partial \overline{P}}{\partial x} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z} + \nu\nabla^2\overline{u}$$

# Turbulent Equations of Motion (y-component)

$$\frac{\partial \overline{v}}{\partial t} + \overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} + \overline{w}\frac{\partial \overline{v}}{\partial z} - f\overline{u} =$$

$$-\overline{a}\frac{\partial \overline{P}}{\partial y} - \frac{\partial \overline{v'u'}}{\partial x} - \frac{\partial \overline{v'v'}}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z} + \nu\nabla^2\overline{v}$$

# Turbulent Equations of Motion (z-component)

$$\frac{\partial \overline{w}}{\partial t} + \overline{u}\,\frac{\partial \overline{w}}{\partial x} + \overline{v}\,\frac{\partial \overline{w}}{\partial y} + \overline{w}\,\frac{\partial \overline{w}}{\partial z} + \vec{g} =$$

$$-\,\overline{a}\,\frac{\partial \overline{P}}{\partial z} - \frac{\partial\,\overline{w'u'}}{\partial x} - \frac{\partial\,\overline{w'v'}}{\partial y} - \frac{\partial\,\overline{w'w'}}{\partial z} + \nu \nabla^{2}\overline{w}$$

# Lesson 10: Part 2

- Stresses
  - – Understand what stresses do to a fluid
  - – Be familiar with the types
    - Viscous Stress
    - Reynolds Stress
- Closure Theory

# Types of Stress

- The force tending to produce deformation in a body
  - Pressure stress
  - Viscous stress
  - Reynolds stress

Shearing stresses

- Shearing Stress: a tangential force per unit area; it is relative to a surface and is in the surface. It occurs when there is a gradient in the velocity field (shear) so that adjacent surfaces of the fluid cannot move simultaneously

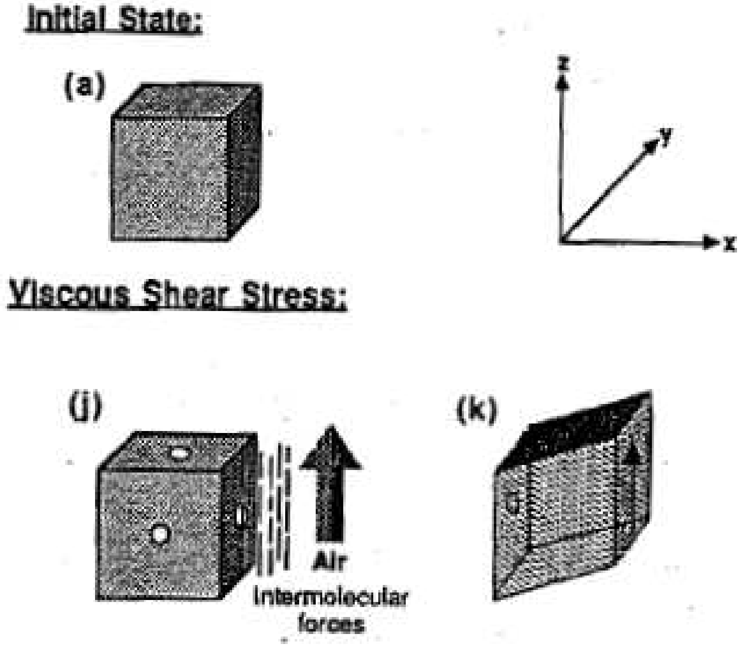
# Types of Stress: Viscous

- <u>Viscous Stress</u>: caused by the viscosity of the fluid as a result of intermolecular forces
  - Viscous stress is proportional to the velocity shear

$$\vec{\tau}_z = \mu\left(\frac{\partial u}{\partial z}\hat{i} + \frac{\partial v}{\partial z}\hat{j}\right)$$

$$\vec{\tau}_x = \mu\left(\frac{\partial v}{\partial x}\hat{j} + \frac{\partial w}{\partial x}\hat{k}\right)$$

$$\vec{\tau}_y = \mu\left(\frac{\partial u}{\partial y}\hat{i} + \frac{\partial w}{\partial y}\hat{k}\right)$$

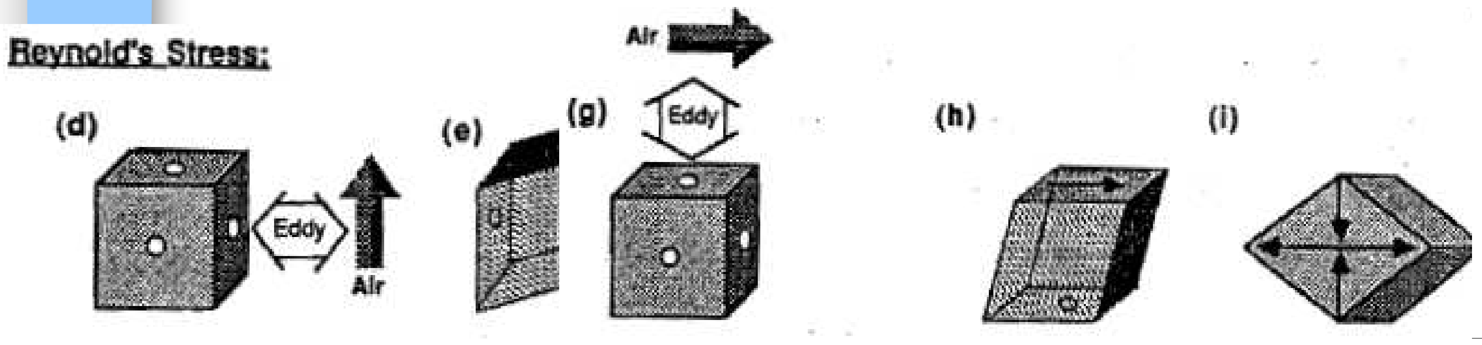


# Momentum Flux

- Momentum (p=m**V**) can be broken into component parts (u, v, w)
- Each velocity can be transferred in any of three directions (x, y, z)
- Result:
  - 9 components of momentum flux
    - i.e. a second order tensor (a matrix)
    - Recall, a scalar is a zero order tensor and a vector is a first order tensor

# Types of Stress: Reynolds

- Reynolds Stress ($\tau_{Reynolds}$): results from the turbulent transport of momentum. The momentum flux is equivalently a stress as it causes *deformation of a fluid parcel*.
  - It is not because of the existence of a real tangential force (as in viscous stress).
  - Reynolds stress is a property of the flow.



# Reynolds Stress

- A *momentum* flux
- Stress term from the Reynolds average of the EOM (right hand side):

$$\tau_{Rxz} = \tau_{Rzx} = -\rho\overline{u'w'},$$

$$\tau_{Ryz} = \tau_{Rzy} = -\rho\overline{v'w'},$$

$$\tau_{Rxy} = \tau_{Ryx} = -\rho\overline{u'v'}$$

# Reynolds Stress Tensor

- To describe Reynolds Stress:

$$\begin{bmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'v'} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'w'} \end{bmatrix}$$

- A second order tensor, which is a symmetric matrix

# Closure (Assumption) Theory

- The problem: we need to express Reynolds stresses as a function of only mean quantities because we cannot directly measure the fluctuations
- Attempt to relate the mean flow to the background flow
  - One way to do this is the *eddy viscosity closure model*

# Eddy Viscosity Closure Model

- Done through analogy with molecular friction (υ), NOT based on physics
- A first order closure scheme
  - The flux terms are approximated using the basic flow properties (minimal assumptions about the basic flow)

- Ekman made further assumptions which we will cover in lesson 11