Wave Equation Procedures in Intermediate Water

Basic wave equation:

\[ L = \frac{g T^2}{2 \pi} \tanh \left( 2 \pi \frac{d}{L} \right) \]

Divide both sides by deep water wave length:

\[ \frac{L}{L_\infty} = \frac{\tanh \left( 2 \pi \frac{d}{L} \right)}{\tanh \left( 2 \pi \frac{d}{L} \right)} = \frac{C}{C_\infty} \]

Multiply both sides by d/L:

\[ \frac{d}{L_\infty} = \frac{d}{L} \tanh \left( 2 \pi \frac{d}{L} \right) \]

Solve for L:

\[ L = L_\infty \tanh \left( 2 \pi \frac{d}{L} \right) \]

Divide both sides by T:

\[ C = C_\infty \tanh \left( 2 \pi \frac{d}{L} \right) \]

Tables (Appendix C of Shore Protection Manual)
if d/L known straightforward to substitute into equation; otherwise if d/ L_\infty known requires iteration
simple with computer and iteration (spreadsheet)

Note that C_O and C_\infty are often used interchangeably, and that the SPM tables use the C_O notation.


1. Find deep water wavelength L_\infty from period T
2. Find deep water speed C_\infty from period
3. Find depth over deep water wave length (d/L_\infty)
4. Enter table to get depth over wave length (d/L) from the value of d/ L_\infty
5. Find wavelength L by dividing known depth by depth over wave length from table
6. Find speed C by dividing wavelength by period
7. Find K_s, shoaling coefficient, also (H/H_\infty)' in table.
8. Find the angle of attack at this depth if \( \alpha_\infty > 0 \), from Snell’s Law.
9. Find the refraction coefficient if \( \alpha_\infty > 0 \); otherwise K_r = 1.
10. Find height of wave
11. Determine if wave has broken (H/d > 0.78)

Example: find L and C of a 10 sec (T) wave in 3 m (d) of water if H_O = 1.5 m and \( \alpha_O = 0 \)

1. \( L_\infty = \frac{g T^2}{2 \pi} = 1.56 \) \( T^2 = 1.56 \times (10)^2 = 156 \) m
2. \( C_\infty = \frac{L_O}{T} = 156 \) m / 10 sec = 15.6 m/sec
3. \( d/L_\infty = 3 \text{m} / 156 \text{m} = 0.0192 \) (should d/L_\infty be greater than 1, the wave is deep water)
4. from table, d/L = 0.056 (which is transitional)
5. \( L = d / (d/L) = 3 \text{m} / 0.056 = 53.2 \) m
   also \( L = L_\infty \tanh(2 \pi d/L) = 156 \times 0.3386 = 52.8 \) m
6. \( C = L / T = 53.2 \text{m} / 10 \text{sec} = 5.32 \text{m/sec} \)
   also \( C = C_\infty \tanh(2 \pi d/L) = 156 \times 0.3386 = 52.8 \text{m/sec} \)
7. \( K_s = (H/H_\infty)' = 1.227 \) (from table)
8. \( H = K_s \times K_r \times H_O = 1.227 \times 1.0 \times 1.5 = 1.841 \) m
10. \( H/d = 1.841 / 3 = 0.614 \), which is less than 0.78, so wave has not yet broken
REFRACTION

\[ \frac{H}{H_o} = K_R \times K_S, \quad H = K_R \times K_S \times H_o \]

\( K_R = \sqrt{\frac{b_o}{b}} = \sqrt{\frac{\cos \alpha_o}{\cos \alpha}} \) = refraction coefficient

- \( b \) = distance between orthogonals
- \( \alpha \) = angle between wave crest and bottom contour, 0 is head on approach, 90 along the beach

Limiting to simple, planar offshore topography

\( K_S = \sqrt{0.5 \times \frac{1}{n} \times \frac{C_o}{C}} = \frac{H}{H_o} \) from \( d/L \) tables.

Example 2, with refraction:

- \( T = 12 \) sec
- \( D = 8 \) m
- \( \alpha_o = 30^\circ \)
- \( H_o = 3 \) m

\[ L_o = 1.56 \times 12 \times 12 = 224.64 \]
\[ C_o = 1.56 \times 12 = 18.72 \]

\[ \frac{d}{L_o} = \frac{8}{224.6400} = 0.0356 \]

from table, \( \frac{d}{L} = 0.0775 \)

\( K_S = 1.092 \)

\[ \tanh(2\pi \frac{d}{L}) = 0.464 \]
\[ L = \frac{d}{L} = 8 / 0.0775 = 103.2258 \]
\[ C = \frac{103.2258}{12} = 8.6022 \]

\[ \sin \alpha = \frac{C}{C_o} \times \sin \alpha_o = \frac{8.60}{18.72} \times \sin 30^\circ = \]
\[ (8.60 / 18.72) \times 0.500 = 0.230 \]

could also use \( \sin \alpha = \tanh(2\pi d/L) \times \sin \alpha_o \)

\( \alpha = \arcsin(0.230) = 13.3^\circ \)

\( K_R = \sqrt{\frac{\cos \alpha_o}{\cos \alpha}} = \sqrt{\frac{\cos 30^\circ}{\cos 13.3^\circ}} = \]
\[ \sqrt{(0.866/0.976)} = \sqrt{(0.887)} = 0.94 \]

\[ H = K_R \times K_S \times H_o = 0.94 \times 1.092 \times 3 = 3.079 \text{ m} \]