What do we mean by EMR?

Propagation through space of a time-varying wave that has both electrical and magnetic components.

Consider a simple sine wave as our model, with:

- Wavelength $\equiv \lambda$ (“lambda”),
- Frequency $\equiv \nu$ (“nu”),
- Speed $V = \nu \lambda$.

The wave’s period $T = \frac{1}{\nu}$.

In a vacuum, the speed is denoted as $c$ ($c \approx 3 \times 10^8$ m/sec).

The real index of refraction $n$ is defined by $n = \frac{c}{V}$.

N.B.: the speed referred to here is that of the waveform, not of any object, so that values of $n < 1$ (and thus $V > c$) are possible.
How do we describe this EMR wave?

Light consists of oscillating electrical fields (denoted $\mathbf{E}$ above), and magnetic fields (denoted $\mathbf{B}$). We’ll concentrate on $\mathbf{E}$ and ignore $\mathbf{B}$, however, we could just as easily describe light using $\mathbf{B}$. We don’t do it because the interaction of magnetic fields with charged particles is more complex than electric fields, but we could.
Light whose electric field oscillates in a particular way is called polarized. If the oscillation lies in a plane, the light is called plane or linearly polarized (top right). Linearly polarized light can be polarized in different directions (e.g., vertical or horizontal above). Light can also be circularly polarized, with its electric field direction spiraling in a screw pattern or helix that has either a right- or left-handed sense (bottom). Seen along propagation axis $x$ this helix has a circular cross-section. Light can also combine linear and circular polarization — its electric field then traces out a helix with an elliptical cross-section. Such light is called elliptically polarized.

We often speak of unpolarized light, yet each individual EMR wave is itself completely polarized. Unpolarized light is actually the sum of light emitted by many different charges that accelerate in random directions. Real detectors like radiometers can only observe the space- and time-averaged intensities of the myriad oscillating charges. If this light has an observable dominant polarization, we call it polarized.

**Polarized light in remote-sensing applications**

![Graph showing polarization by reflection by water (n = 4/3)]

Most environmental light sources such as sunlight are unpolarized. Thus passive remote-sensing systems usually don’t benefit from the extra information that a polarized light source provides. Radar and other active remote-sensing systems usually emit

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polarized EMR, and so they can exploit the different signal patterns reflected when different polarizations illuminate the surface.

Yet even reflected sunlight can be polarized, as the graph above indicates. It shows how polarization varies with **incidence angle** $\theta$ (incident EMR’s angle with the surface normal) for unpolarized light reflected by a smooth water surface. Degree of polarization $P$ varies from 0 (completely unpolarized reflected light) to 1 (completely polarized reflection). In fact, the graph explains an entire industry — manufacture of polarizing sunglasses.

Like all linear polarizers, the plastic sheet polarizers used in some sunglasses have a particular direction along which linearly polarized light is *absorbed least*; polarized light is absorbed more strongly in other directions. This minimum-absorption direction is called the **polarizer’s transmission axis**. When linearly polarized light’s oscillation direction parallels the polarizer transmission axis, we get maximum transmission. Rotate the light source 90° about its propagation axis (or rotate the filter 90° about that axis), and we get minimum transmission.

- At what $\theta$ is there maximum polarization for reflection by water?
- How does the $P(\theta)$ graph help explain the changing effectiveness of polarizing sunglasses?
- To be most effective in reducing reflected glare, should the sunglasses’ transmission axis be oriented horizontal, diagonal, or vertical to the water surface? (Hint: Examine the drawing above to see the oscillation direction of the reflected, partially polarized light.)

Now return to our explanation of EMR’s wave nature. Mathematically, the wave amplitude $E$ is given by, at any time $t$ and position $x$ (with $x$ measured along the direction of propagation):

$$E = E_0 \sin(2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right])$$

(Eq. 1.)
By setting $t, x$ to fixed values in Eq. 1, we can determine the electromagnetic wave’s propagation direction.

In addition, we can use Fourier analysis to reduce any waveform, no matter how complicated, to a sum of simple sine and cosine waves. This has mathematical advantages when we try to analyze the wave’s physical significance and sources.

Note that Eq. 1 can be rewritten as:

$$E = E_0 \sin\left(\frac{2\pi}{\lambda} [ct - x]\right) \rightarrow$$

$$E = E_0 \sin(2\pi \nu [t - x/c]) \rightarrow$$

(factor out $c$ from the $[ ]$, and note that $c/\lambda = \nu$)

$$E = E_0 \sin(\omega t - \delta),$$

where $\omega = 2\pi \nu = 2\pi/T$ and $\delta = 2\pi x/\lambda$.

Note that $\omega$ is described as the **rotational angular frequency** (in radians/second), and $\delta$ denotes a **phase angle** (where “phase” refers to angular separation).

Remember that 1 radian (~ $57^\circ$) is that portion of a circle equal to the circle’s radius, so converting to radian notation above scales the x-propagation in “natural” rotational (or wave) terms.

What is the source of EMR?

Fundamentally, EMR’s source is the acceleration of electrical charges. A convenient example of such a charge is an electron.

If one electron approaches another, their paired negative charges will result in mutual repulsion. The closer they approach one another, the stronger this repulsive force (and the associated charge accelerations) will be.
Remember that Coulomb’s law describes this force $F$ between a pair of charges $q_1$ and $q_2$ by:

$$F = \frac{k q_1 q_2}{R^2}.$$  

So the repulsive force varies inversely with the square of the charge separation $R$. The **electric field strength** vector ($\mathbf{E}$) indicates the force per unit charge so that $\mathbf{E} = \frac{F}{q}$. Physically, the electric field lines are defined by the force lines.

If we insert an electron into a previously vacant region of space, the charge’s presence is communicated at speed $c$ along the electric field lines. By accelerating the electron to a new position, we have altered the field lines, producing bends or kinks in them. Because the kinks in the electric field lines propagate at the finite speed $c$ (in a vacuum), we say that the radiation propagates at speed $c$.

Back and forth acceleration of the electron produces oscillating field lines or **waves**. Note, however, that along the line of oscillation, there is no detectable displacement (and hence no electromagnetic wave).
The time-averaged magnitude (or amplitude) $E$ of the electric field strength $E$ varies as:

$$E = \frac{qa}{R} \sin(\theta),$$

where $\theta$ is the angle between the direction of the oscillation and the direction of our detector, $a$ is the magnitude of the electron acceleration, and $R$ is the distance over which the acceleration occurs. In the diagram above, the $z$-axis corresponds to $\theta = 0^\circ$, while the $x$-axis and $y$-axis (in fact, the whole $x$-$y$ plane) corresponds to $\theta = 90^\circ$.

Now the intensity (denoted $I$, a measurable electromagnetic quantity) of this EM wave varies as the square of the electrical field’s magnitude. Thus $I \propto E^2 \sin^2(\theta)$.

We’ve just described a transmitter/emitter, in which we somehow accelerated the electrons. Conversely, if an oscillating wave “washes by” a charge that is initially at rest, the charge will accelerate in response. In this case, the electromagnetic wave does work on the charge. Now we have a receiver in which the originally stationary charge is accelerated by the presence of EMR.

For convenience sake, thus far we have described EMR using wave terminology. This is appropriate because EMR often displays wavelike characteristics; e.g., interference. However, later it will be more convenient to discuss EMR using particle (or photon) terminology, as when we discuss scattering.

Photons’ (massless) kinetic energies $E$ are described by:

$$E = h \nu = h \frac{c}{\lambda},$$

where $h = 6.626 \times 10^{-34}$ J sec, Planck’s constant. How does $E$ depend on both frequency and wavelength?
How do we describe naturally occurring EMR?

There are two main classes of descriptions that we need: 1) geometric, and 2) spectral. The first class tells us how EMR is affected by the distance between source and receiver, and by their relative sizes and orientations. The second group of descriptions tells us how the intensity of EMR depends on $\nu$ (or $\lambda$). This is important since most natural EMR sources have very broad power spectra.

1) geometry and EMR

We begin with some definitions of radiometric quantities: radiant energy, radiant flux, radiant intensity, radiant exitance, irradiance, and radiance.

However, we begin with an even more basic, purely geometric definition, that of solid angle. We define a planar angle (OCB below) as the ratio of the circular arc length $s$ to that of its corresponding radius (line OB of length $R$). The resulting dimensionless ratio is given the units of radians, and can be thought of as a measure of the set of directions in OCB.

By analogy, a solid angle is defined as a measure of the set of directions radiating from a point (O above) and ending at the surface of a sphere whose center is O.

Since the area $A$ of the sphere’s surface in which we’re interested has units of length$^2$, it makes sense that the dimensionless solid angle is calculated by dividing $A$ by the square of the sphere’s radius $R$. Solid angles $\omega$ have units of steradians (sr).

Mathematically, the differential solid angle $d\omega$ is given by:

$$d\omega = \frac{dA}{R^2} \quad \text{(Eq. 2)}$$

If $R \neq f(A)$, then $\omega = A/R^2$ \{i.e., “$\omega = A/R^2$ if R does not vary over A”\}. This equation is exact only when $A$ lies on the surface of a sphere and $R$ is measured from the sphere’s center. Now, $\omega \approx A/R^2$ for the sun’s solid angle as seen from the earth. However $\omega \approx A/R^2$ is not a good assumption when we try to measure, say, the solid angle of the ceiling in this room (where the distance $R$ to a point directly overhead is much less than that to the room’s corners).

Now, using our formulas above, what’s the solid angle (as seen from the earth’s surface) of a 1 km-radius circular cloud whose altitude = 1 km?

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If \( \omega = A/R^2 \) here then \( \omega \) should equal \( \frac{\pi \times 1 \text{ km}^2}{1 \text{ km}^2} = \pi \approx 3.14 \text{ sr} \). In fact, however, the correct answer is about half this value. To calculate solid angle accurately in these “close-up” conditions (where \( R \) does vary with angle), we must integrate Eq. 2 over solid angle. This in turn, requires redefining \( d\omega \) in terms of polar coordinates.

In polar coordinates, \( dA = R^2 d\omega \) is an incremental spherical surface area. Specifically, \( Rd\theta \) (incremental change in \( dA \)’s sides as we move along zenith angle \( \theta \)) times \( R\sin(\theta)d\phi \) (incremental change in \( dA \)’s top and bottom as we move along azimuth angle \( \phi \)) yields \( d\omega \). In other words:
\[
d\omega = \frac{dA}{R^2} = d\phi \sin(\theta)d\theta
\]

Returning to the circular cloud problem, note that \( \phi \) ranges from 0 to \( 2\pi \) (azimuth turns throughout a complete circle as we trace the edge of the cloud), but \( \theta \) varies only between zenith angles of 0 and \( \pi/4 \) (we look from the zenith down toward an angle 45° from the zenith). So now

\[
\omega = \int d\omega = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi/4} \sin(\theta)d\theta = 2\pi \left[-\cos(\pi/4) + \cos(0)\right],
\]

\[
\omega = 2\pi \left[1 - 0.707\right] \approx 1.84 \text{ sr} \text{ (versus our incorrect 3.14 sr earlier)}.
\]

Remember, however, that if \( R \neq f(A) \), then \( \omega = A/R^2 \) is exactly true. Take, for example, a problem with immediate relevance to satellite imaging, that of calculating the solid angle of the sun as seen from earth. From the earth’s surface, the sun’s (planar) angular diameter \( \sim 1/107 \text{ radians} \).
Denote the earth-sun distance as \( R_{es} \) and note that \( A = \pi R_{sun}^2 \). Now \( \omega = \frac{\pi R_{sun}^2}{R_{es}^2} \). But stating that the sun’s angular diameter \( \sim 1/107 \) radians is the same as saying that \( R_{sun}/R_{es} = \left(\frac{1}{2}\right) \frac{1}{107} \). This ratio makes \( \omega = \frac{\pi}{(2 \times 107)^2} = 6.86 \times 10^{-5} \text{ sr} \).

We can appreciate how angularly small the sun is from earth (or from an earth-orbiting satellite) when we consider that a sphere subtends a solid angle of \( 4\pi \text{ sr} \) \( \omega = \frac{4\pi R^2}{R^2} = 4\pi \), angularly more than \( 183,000 \) times as large as the sun). That all EMR of any significance for meteorology and oceanography comes from such a small region of the sky is remarkable.

In these two problems, we have distinguished between a point source of EMR \( (< 1/20 \text{ radian or } \sim 3^\circ) \) and an extended source \( (\geq 1/20 \text{ radian}) \). This distinction will be important when we consider the radiometric units of radiance (for point sources) and irradiances (for extended sources).

### Radiometric Definitions:

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>mathematical form</th>
<th>symbol</th>
<th>SI units</th>
</tr>
</thead>
<tbody>
<tr>
<td>radiant energy</td>
<td>( Q )</td>
<td>( Q )</td>
<td>J (joule)</td>
</tr>
<tr>
<td>spectral radiant energy</td>
<td>( \frac{dQ}{d\lambda} )</td>
<td>( Q_\lambda )</td>
<td>( \frac{J}{m} ) (joule/meter)</td>
</tr>
<tr>
<td>spectral radiant energy</td>
<td>( \frac{dQ}{dv} )</td>
<td>( Q_v )</td>
<td>J sec (joule sec)</td>
</tr>
<tr>
<td>radiant flux (or power)</td>
<td>( \frac{dQ}{dt} )</td>
<td>( \Phi ) (“phi”)</td>
<td>( \frac{J}{sec} ) or Watt</td>
</tr>
<tr>
<td>radiant intensity</td>
<td>( \frac{d\Phi}{d\omega} )</td>
<td>( I )</td>
<td>( \frac{W}{sr} )</td>
</tr>
<tr>
<td>radiant exitance</td>
<td>( \frac{d\Phi}{dA} )</td>
<td>( M )</td>
<td>( \frac{W}{m^2} )</td>
</tr>
<tr>
<td>irradiance</td>
<td>( \frac{dE}{dA} )</td>
<td>( E )</td>
<td>( \frac{W}{m^2} )</td>
</tr>
<tr>
<td>radiance</td>
<td>( \frac{dE}{\cos(\theta) d\omega} ) or ( \frac{d^2\Phi}{dA d\cos(\theta) d\omega} )</td>
<td>( L )</td>
<td>( \frac{W}{m^2 \text{ sr}} )</td>
</tr>
</tbody>
</table>

(After Table 6.4 in Robinson)

Our table shows how several commonly used radiometric quantities are derived from the fundamental one, the radiant energy \( Q \). Note that the spectral quantities are
ideally monochromatic \(i.e.,\) single-wavelength or -frequency). In practice, however, they are defined over a small but finite spectral range. Why?

In other words, in reality a measurable \( Q = \int_{\lambda_1}^{\lambda_2} Q_\lambda(\lambda) \, d\lambda \) and \( Q_\lambda \) is a definable but unmeasurable quantity. Let’s consider some of the other definitions.

**Radiant flux** (or power, \( \Phi \)) leads to a pair of quantities that differ only in their assumed orientation rather than in their mathematical definitions. First is the (hemispheric) **radiant exitance** \( M \), the energy flux *leaving* a (plane) surface and traveling into \( 2\pi \) steradians. Clearly the reverse can happen, so it’s useful to define a complement to this, the (hemispheric) **irradiance** \( E \), which is the energy flux *arriving* at a (plane) surface from \( 2\pi \) steradians.

We qualify these definitions as *hemispheric* because it’s possible for irradiance to arrive from more than \( 2\pi \) steradians, and obviously radiators can emit into solid angles larger than a hemisphere (common examples?). However, the definitions were devised because in many locales and for some instruments, a hemisphere is the largest possible field of view.

Irradiance is a very crude directional measure of radiant energy fluxes, so the **radiance** \( L \) is used to “pinpoint” directional variability in an EMR environment. Now we put our definition of solid angle \( \omega \) to work.

Imagine a small detector with a very small field of view, say of point source size \( (~3^\circ) \). Human foveal vision is a good example of such a detector. EMR enters this small solid angle from some source and illuminates the detector. Clearly the irradiance \( (E \text{ in } \text{W m}^{-2}) \) around the detector partly determines the signal strength, but so too does the detector’s field of view \( (d\omega) \). The larger this is, the greater the signal.

In addition, the orientation of the detector’s surface normal determines how effective an EMR receiver it is. If the detector is *parallel* to the EMR’s propagation direction \( (i.e., \text{the source’s zenith angle } \theta = 90^\circ) \), then the signal strength is *zero*.  

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Conversely, if the detector is perpendicular to the EMR (θ = 0˚), the received signal strength is at a maximum.

The **cosine law** describes the relationship between irradiance E received at angle θ and the maximum irradiance E_0 at normal incidence (θ = 0˚):

\[ E(\theta) = E_0 \cdot \cos(\theta) \] (Eq. 3)

Unlike irradiance E, we define radiance L so that it’s *independent of detector orientation*.

So three factors (E, θ, ω) determine this narrow field-of-view EMR measure, as our earlier definitions indicated (\( L = \frac{dE}{\cos(\theta)} d\omega = \frac{d^2\Phi}{dA \cos(\theta) d\omega} \)). Note in particular that we’re interested in the detector’s surface area as *projected* on the converging bundle of photons, thus the factor of \( \frac{1}{\cos(\theta)} \). Provided we know θ, this factor eliminates L’s dependence on detector orientation.

Why bother to define so complicated a quantity? First, as our example of foveal vision indicated, radiance is a good analog of the way we (and other narrow field-of-view detectors) measure EMR. Many satellite radiometers and cameras measure radiances rather than irradiances.

The second reason for defining radiance as we have is that, unlike irradiance, radiance does not change with distance (strictly true only in a vacuum and for extended EMR sources). This invariance means that radiance detectors are not misled by changing distances between them and light sources. The evolutionary advantages of this are fairly obvious, and it makes the difficult task of analyzing satellite images easier.

But how does the invariance of radiance work physically?
First, how does irradiance change with distance? Take the sun as our example of a more-or-less constant emitter of radiant flux \( \frac{dQ}{dt}, \text{Watts} \). As the diagram below indicates, this flux radiates into an ever-larger volume of space bounded by a sphere of ever-larger surface area (which increases as \( \text{radius}^2 \)).

Thus it makes sense that the constant solar radiant flux is spread out over an increasingly large surface as we get further from the sun. Mathematically, the flux per unit area (i.e., irradiance) \textit{decreases inversely with the square of distance} because that’s how fast our imaginary sphere’s surface area is increasing. This yields the \textbf{inverse-square law} which relates the irradiances \( E(d) \) at distances \( d_1 \) and \( d_2 \) by:

\[
E(d_2) = E(d_1) \times \left(\frac{d_1}{d_2}\right)^2
\]  
(Eq. 4).

But if the irradiance falling on our detector decreases as \( \frac{1}{\text{distance}^2} \), why is radiance, which is depends on irradiance, unchanged? The answer is that, although the irradiance reaching our detector decreases as \( \frac{1}{\text{distance}^2} \), \textit{so does the sun’s apparent size as seen at the detector}. 

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In other words, our detector occupies a smaller and smaller fraction of the imaginary sphere’s surface area, and in fact the reduction scales exactly as $\frac{1}{\text{distance}^2}$. The net result is that radiance (analogous to brightness) will not vary with distance.

In general, both emitted and reflected radiance $L$ depend on the zenith angle $\theta$. A good example of this dependence is **specular** or mirrorlike reflections from smooth (or quasi-smooth) surfaces, something seen as ocean **sun glint** in satellite images.

The extreme opposite of a specular surface is a **Lambertian** (or diffusely reflecting) surface, one for which $\theta$ has no effect on the reflected $L$, even if the light source itself is not diffuse. A sunlit snow pack or surface covered with matte-finish paint are examples that approach this ideal.

Now, if $L \neq f(\theta)$, a simple relationship exists between this Lambertian surface’s exitance ($M$) and the radiance illuminating it. In general, remember, this relationship is complicated and the differential exitance $dM = L \cos(\theta) d\omega$ will vary with both $L$ and $\theta$. However, in the Lambertian case,

$$M = \int_{\phi=2\pi} dM = \int_{\phi=2\pi} \int_{\theta=\pi/2} L \cos(\theta) \sin(\theta) \cos(\theta) \sin(\theta) d\theta d\phi = \int_{\theta=0}^{\pi/2} L \int_{\phi=0}^{2\pi} \cos(\theta) \sin(\theta) d\phi d\theta = 2\pi L \int_{\theta=0}^{\pi/2} \sin^2(\theta) d\theta = 2\pi L \left[ \frac{\sin^2(\pi/2)}{2} - 0 \right] = \pi L.$$

This result may be surprising — since there are $2\pi$ steradians in a hemisphere, why should a constant $L$ (measured in Wm$^{-2}$sr$^{-1}$) yield an exitance only $\pi$ times larger than $L$?

Consider what’s occurring physically, however. The power emitted by a planar surface element is being mapped into a hemisphere, and not all $L$ will contribute equally to the hemispheric $M$ measured above the surface.

Similarly, if we have a directionally uniform or **isotropic** radiance field with $L \neq f(\theta)$, not all $\theta$ will contribute equal $L$ to the planar receiving surface: each is weighted by $\cos(\theta)$, yielding $E = \pi L_{\text{isotropic}}$. 

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Consider another more practical problem. The sun is too bright to look at because its radiance is so high, but snow’s is not. Why? Neglecting transmission losses through the atmosphere, \( L_{\text{sun}} = E_{\text{sun}}/\omega_{\text{sun}} = \frac{1380 \text{ Wm}^{-2}}{6.86 \times 10^{-5} \text{ sr}} = 2.01 \times 10^{7} \text{ Wm}^{-2}\text{sr}^{-1} \).

If snow is a perfect Lambertian reflector, what is the relationship between \( L_{\text{sun}} \) and \( L_{\text{snow}} \)? Saying that snow is a perfect reflector means \( M_{\text{snow}} = E_{\text{snow}} \) or snow’s exitance = snow’s irradiance. In other words, \( \pi L_{\text{snow}} = L_{\text{sun}} \omega_{\text{sun}} = E_{\text{sun}} \) and so:

\[
L_{\text{snow}} = \frac{E_{\text{sun}}}{\pi} = \frac{1380 \text{ Wm}^{-2}}{\pi} = 439 \text{ Wm}^{-2}\text{sr}^{-1},
\]

and in general \( \frac{L_{\text{snow}}}{L_{\text{sun}}} = \frac{\omega_{\text{sun}}}{\pi} = 2.19 \times 10^{-5} \) for a Lambertian surface.

This explains why the radiance of highly reflecting snow is bearably bright, but the sun is not (we’ve ignored a factor of \( \cos(\theta) \) times \( E_{\text{sun}} \) that accounts for the effect of the sun’s elevation on the actual \( E_{\text{sun}} \)).

Finally, we consider some EMR bookkeeping. If we allow that EMR is conserved (i.e., not transformed into matter), what can happen to it? Given that the probability of something happening to EMR is unity (1), its possible fates are that:

a) some fraction \( r \) of the EMR may be reflected \( (r \equiv \text{reflectance}) \),
b) another fraction \( \alpha \) may be absorbed \( (\alpha \equiv \text{absorptivity}) \),
c) in transparent media, some fraction \( t \) will be transmitted \( (t \equiv \text{transmissivity}) \).

Mathematically, conservation of energy says that:

\[
 r + \alpha + t = 1. \quad (\text{Eq. 5})
\]
Note too that the spectral energy reflected by a material is the product or convolution of its reflectance $r_\lambda$ and the incident energy. Thus

$$E_{r,\lambda}(\theta) = \cos(\theta) \cdot r_\lambda \cdot E_{\text{inc},\lambda} \quad \text{(Eq. 6)}$$

where $E_{\text{inc},\lambda}$ is the irradiance illuminating the material (itself a function of distance; see Eq. 4) and the factor of $\cos(\theta)$ is from the cosine law (see Eq. 3). Below is a graph of one such convolution for a slide-projector lamp and a sample of white paint. Based on what you see in the graph, predict the illuminated paint’s color. What problem is there with this purely spectral analysis?

![Graph of spectral energy convolution](image)

2) spectral variability of EMR

Many (but not all) of the EMR sources that will concern us in satellite remote sensing are referred to as (approximate) blackbody radiators. A blackbody is defined as matter that completely absorbs all EMR incident on it, regardless of the radiation’s $\lambda$, incidence angle, and polarization. No real radiator completely fits this description.

Most useful to us, a blackbody and its approximate, but real, cousins have EMR emission spectra that are determined only by their temperatures. In other words, if you...
can measure a blackbody’s EMR spectrum, you know its temperature no matter how distant it is.

Can blackbody radiation ever occur in nature? While perfectly black (i.e., perfectly absorbing) objects are hypothetical, blackbody radiation occurs whenever enclosed spaces are in thermal equilibrium. Imagine a vacuum bottle (or any insulated space) made of any material whatsoever. Bohren’s (1987) example of aluminum, which is highly reflective (rather than absorbing) over much of the EMR spectrum indicates how arbitrary we can be.

Now we introduce a slab of aluminum into this real vacuum bottle and seal it. When the bottle’s temperature is constant with time, we say it’s in thermal equilibrium. By definition, the slab is in thermal equilibrium too. What is the nature of the radiation in the bottle? If the slab is in thermal equilibrium, the rate at which it absorbs EMR must equal the rate at which it emits EMR (otherwise its temperature would change).

By analogy to absorptivity $\alpha$, we define an object’s emissivity $\varepsilon$ as its emission rate relative to that of a blackbody at the same temperature.

**Kirchhoff’s law:** For a given wavelength, polarization state, and direction, a body’s emission and absorption are equal. For real bodies, the rates of emission and absorption will be less than that of a blackbody at the same temperature.

Literally, Kirchhoff’s law says that $\alpha_\lambda = \varepsilon_\lambda$. Very loosely translated, Kirchhoff’s law is “good absorbers are good emitters,” but Bohren (1987, 1991) discusses some of the conceptual problems this causes (e.g., can an internally heated blackbody (a “good emitter”) in deep space be a “good absorber” of EMR that is not incident on it?).
Now back to our aluminum vacuum bottle. What kind of radiation fills its interior when it’s in thermal equilibrium? By the definition of thermal equilibrium, the aluminum bottle’s emission spectrum must be identical to its absorption spectrum.

If we imagine a blackbody in the bottle, then its assuredly blackbody radiation spectrum must also be the same as the real radiation field filling the interior. Replacing the blackbody with the aluminum slab changes nothing: blackbody radiation bathes them both, even though the aluminum is far from a perfect absorber.

What is the aluminum’s emissivity? Because aluminum is opaque in both the visible and the infrared (i.e., transmissivity \( t = 0 \)), Eq. 5 may be rewritten as:

\[
\alpha_{\text{opaque}} = 1 - r_{\text{opaque}},
\]

meaning that, because \( r \sim 1 \) for aluminum, its absorptivity is small. If aluminum’s \( \alpha \) is small, then so is its \( \varepsilon \).

The secret to achieving blackbody radiation was in specifying an insulated enclosure. Now the walls’ high \( R \) (and low \( \varepsilon \)) becomes an asset, repeatedly reflecting that EMR which wasn’t absorbed and making thermal equilibrium possible.

**spectral properties of blackbody radiation**

Starting from the kinetic theory of temperature, we can show that the spectral blackbody radiance \( B_\lambda \) is given by:

\[
B_\lambda = \frac{2hc^2}{\lambda^5 \left( \exp(\frac{hc}{\lambda kT}) - 1 \right)}, \quad (\text{Eq. 7})
\]

where \( T \) is temperature (in Kelvins), \( h \) is Planck’s constant, \( c \) is the speed of light, and \( k = 1.38 \times 10^{-23} \) Joule K\(^{-1}\), Boltzmann’s constant. Equation 7 is called Planck’s law, and over much of a blackbody’s spectrum we can accurately say that:

\[
B_\lambda \propto \lambda^{-5} \exp(-\frac{1}{\lambda T}).
\]

This Planckian energy distribution produces a characteristic peaked shape similar to the visible solar spectrum. Note that the magnitude of \( B_\lambda \) depends on both wavelength and temperature, although not in any immediately obvious way.
Using our approximation for Planck’s law \( B_\lambda \propto \lambda^{-5} \exp(-1/\lambda T) \), we can derive an equation for the wavelength at which any blackbody radiance spectrum peaks. This equation is called \textbf{Wien’s law}, and is:

\[
\lambda_{\text{max}} = \frac{2898 \mu\text{m K}}{T}, \quad \text{(Eq. 8)}
\]

where the temperature \( T \) is in Kelvins. Solving for \( T \) gives us an approximate way of estimating a real radiator’s temperature from its emission spectrum.

For example, the maximum of the solar spectrum is at \( \sim 0.475 \mu\text{m} \) (microns). By reworking Wien’s law, we get the sun’s \textbf{color temperature} \( T_c \):

\[
T_c = 2898 \mu\text{m K}/0.475 \mu\text{m} \sim 6100 \text{ K}.
\]

In the lighting industry, “color temperature” is a shorthand for a real light’s unknown emission spectrum based on the known spectrum of an unreal radiator, a blackbody.

Note that as a radiator’s mean temperature increases, so does the frequency \( \nu \) of its maximum output \( (\nu_{\text{max}} = c \frac{T}{2898 \mu\text{m K}} ) \). In other words, the entire spectrum...
shifts toward shorter wavelengths as $T$ increases. Where would the emission maxima of the sun and the earth occur relative to each other?

The integrated blackbody radiance $B$, is defined by $B = \int_0^\infty B_\lambda \, d\lambda$, although this hardly seems helpful at first. However, we can substitute Eq. 7 for $B_\lambda$ and solve for $B$ analytically. If we do, integration by parts yields:

$$B = \frac{2\pi^4}{15} \frac{k^4}{c^2} \frac{T^4}{h^3} = \frac{\sigma}{\pi} T^4,$$

where we define the **Stefan-Boltzmann constant** $\sigma$ as:

$$\sigma = \frac{2\pi^5}{15} \frac{k^4}{c^2} \frac{h^3}{c^2} = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}.$$

Note that $B$ is a blackbody *radiance*; often we want a blackbody irradiance $E_{BB}$ or a blackbody exitance $M_{BB}$. Assuming that the blackbody radiates isotropically (as do its real counterparts), we get the **Stefan-Boltzmann relationship**:

$$M_{BB} = E_{BB} = \pi B = \sigma T^4,$$  \hspace{1cm} (Eq. 9)
where $T$ is the blackbody (or radiant) temperature in Kelvins. If emissivity $\varepsilon = 1$, then $T_{\text{rad}} = T_{\text{kin}}$, the ordinary sensible (or kinetic) temperature. \{Note that when we measure sea-surface temperatures (SST), Eq. 9’s $T_{\text{rad}}$ = the sensible temperature of only the water column’s top 1 mm.\} Because $\varepsilon < 1$ for all real materials, $M_{\text{BB}} = \varepsilon \sigma (T_{\text{kin}})^4$. If we don’t know $\varepsilon$ initially, then we must: 1) assume that the remotely measured $M_{\text{BB}} = \sigma (T_{\text{rad}})^4$, 2) measure $T_{\text{kin}}$ directly, then 3) calculate $\varepsilon = \left[ \frac{T_{\text{rad}}}{T_{\text{kin}}} \right]^{4}$. Once we know $\varepsilon$ for a given material, we can calculate $T_{\text{kin}}$ from remotely measured $M_{\text{BB}}$.

Even with such basic equations, we can solve useful remote-sensing problems.

1) What is the sun’s **equivalent blackbody temperature**, the effective temperature of the sun’s apparent surface, given only the earth’s solar constant $E_{\text{sun}}$ and the sun’s solid angle $\omega_s$? (For now, assume $\varepsilon = 1$.)

   We know that:  
   a) radiance (whether $L$ or $B$) does not depend on distance, 
   b) the sun’s blackbody exitance $M_{\text{BB}}$ is $\pi$ times its radiance, 
   c) $M_{\text{BB}} = \sigma (T_{\text{sun}})^4$, 
   d) $\omega_{\text{sun}} = \frac{\pi}{(214)^2}$.

Start by calculating $B_{\text{sun}} = E_{\text{sun}}/\omega_{\text{sun}} = \frac{(214)^2 \times 1380 \text{ Wm}^{-2} \pi \text{ sr}}{ \pi \text{ sr}} = 2.01 \times 10^7 \text{ Wm}^{-2}\text{sr}^{-1}$.

Now $M_{\text{BB}} = \pi B_{\text{sun}} = 6.32 \times 10^7 \text{ Wm}^{-2}$,
and $T_{\text{sun}} = \left[ \frac{M_{\text{BB}}}{\sigma} \right]^{1/4} = \left[ \frac{6.32 \times 10^7 \text{ Wm}^{-2}}{5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}} \right]^{1/4} = 5778$ Kelvins.

Note how we have built on the fundamental ideas developed earlier in order to answer a very practical question. Note also that the sun’s color temperature (6100 K) is higher than its equivalent blackbody temperature. Why?

Remember that the color temperature was calculated using the most energetic wavelength in the solar spectrum, while $M_{\text{BB}}$ is the result of an integral over all $B_{\lambda}$ in the sun’s output, and these $B_{\lambda}$ are by definition less energetic than $B_{\lambda, \text{max}}$.

2) What is the surface temperature of the moon ($T_m$) given that the moon’s average reflectance $r$ (also called the albedo) is 7%? What are some possible problems with our estimate of $T_m$?

Prof. Raymond Lee; SO431; EMR basics for remote sensing
Start by assuming (accurately) that the earth’s and moon’s solar constants are the same (i.e., \( E_{\text{sun}} = E_{\text{moon}} = 1380 \text{ Wm}^{-2} \)) and that we can ignore any atmospheric influence on \( T_m \). Again, we assume that the sunlit side of the moon radiates isotropically as a blackbody.

Our approach is, as earlier, to use energy conservation. In other words, assuming thermal equilibrium, the irradiance not reflected (i.e., absorbed) by the moon’s sunlit hemisphere equals its spherical exitance (where the appropriate surface area is the moon’s, \( 4\pi (R_{\text{moon}})^2 \)). Symbolically,

\[
(1 - \text{albedo}) * E_{\text{moon}} * (\text{projected sunlit area}) = M_{\text{moon}} * (\text{moon’s area}),
\]

and mathematically,

\[
(1 - r_{\text{moon}}) * E_{\text{moon}} * \pi(R_{\text{moon}})^2 = M_{\text{moon}} * 4\pi(R_{\text{moon}})^2,
\]

which can be solved for \( M_{\text{moon}} = \frac{(1 - r_{\text{moon}}) * E_{\text{sun}}}{4} = 321 \text{ Wm}^{-2} \). Next we calculate:

\[
T_m = \left[ \frac{M_{\text{moon}}}{\sigma} \right]^{1/4} = 274 \text{ Kelvins}.
\]

Our too-large answer ignores the moon’s slow rotation and thus the uneven exposure times encountered over its sunlit side at any given moment. Also, the lack of a lunar atmosphere makes heating of the moon’s surface even more uneven.

Both realities complicate our theoretical calculation, which is based on simple geometry and on the behavior of isotropic blackbody radiators.

Consider another very crucial satellite remote sensing problem: determining the potential for global warming. We will show that measuring changes in atmospheric absorptivity \( \alpha_{\text{atm}} \) gives us an approximate \( \Delta T_{\text{earth}} \). What happens to the earth’s average temperature if we measure an \( \alpha_{\text{atm}} \) increase of, say, 2%? (Assume that this increase in \( \alpha_{\text{atm}} \) also increases \( \varepsilon_{\text{atm}} \) by 2%).

We start by drawing a schematic of our earth-atmosphere system. Note that the moon’s exitance \( M_{\text{moon}} = \frac{(1 - r) * E_{\text{sun}}}{4} \) can be used to calculate \( M_{\text{earth}} \) if we substitute the earth’s average albedo of \( r = 0.3 \).
We assume a constant-density atmosphere of finite thickness, which is acceptable for our purposes here. This atmosphere is not a blackbody (i.e., $\varepsilon_{\text{atm}} < 1$).

Qualitatively, we say that at any altitude above an earth in thermal equilibrium:

irradiance absorbed by the earth from the sun + atmosphere = exitance of the earth + atmosphere

Mathematically, this becomes:

\[
\frac{(1 - r_{\text{earth}})E_{\text{sun}}}{4} + \varepsilon_{\text{atm}}\sigma T_{\text{atm}}^4 = \sigma T_{\text{earth}}^4
\]

(absorbed solar) + (↓ atm. emission) = (↑ earth’s surface emission)

and

\[
\frac{(1 - r_{\text{earth}})E_{\text{sun}}}{4} = (1 - \varepsilon_{\text{atm}})\sigma T_{\text{earth}}^4 + \varepsilon_{\text{atm}}\sigma T_{\text{atm}}^4
\]

(absorbed solar) = (% transmitted ↑ earth’s surface emission) + (↑ atm. emission)
Note that both at the top and bottom of the atmosphere \( \frac{(1 - r_{\text{earth}})E_{\text{sun}}}{4} \) is the solar energy input that drives the earth-atmosphere system and \( T_{\text{earth}} \) is the earth’s surface temperature (in Kelvins).

Now we add our two equations above to get:

\[
\frac{(1 - r_{\text{earth}})E_{\text{sun}}}{2} = (2 - \varepsilon_{\text{atm}})\sigma T_{\text{earth}}^4 , \quad \text{or}
\]

\[
\varepsilon_{\text{atm}} = 2 - \frac{(1 - r_{\text{earth}})E_{\text{sun}}}{2\sigma T_{\text{earth}}^4} \quad \text{(Eq. 10)}.
\]

If \( E_{\text{sun}} = 1380 \text{ Wm}^{-2} \), \( T_{\text{earth}} = 287 \text{ Kelvins} \), and \( r_{\text{earth}} = 0.3 \), then \( \varepsilon_{\text{atm}} \approx 0.74 \).

In other words, over the EMR spectrum, the earth’s atmosphere has a high (but not the highest possible) \( \varepsilon \) and \( \alpha \). What if \( \varepsilon \) increases to 0.76? Solving Eq. 10 for \( T_{\text{earth}} \), we get:

\[
T_{\text{earth}} = \left[\frac{(1 - r_{\text{earth}})E_{\text{sun}}}{2(2 - \varepsilon_{\text{atm}})\sigma}\right]^{1/4}
\]

and substituting \( \varepsilon_{\text{atm}} = 0.76 \) yields \( T_{\text{earth}} \approx 287.9 \text{ Kelvins} \). So a relatively modest increase in atmospheric absorptivity (and thus emissivity) produces nearly a 1˚C temperature rise.

We do not know:

1) whether \( \Delta\varepsilon_{\text{atm}} = +0.02 \) would actually occur in response to current increases in \( \text{CO}_2 \) concentrations, or
2) even if it did, whether there would be negative feedback mechanisms affecting \( T_{\text{earth}} \).

We do know that \( \Delta T_{\text{earth}} = +1.0˚ \text{ C} \) would be wrenching economically and politically, if not physically.