EMR’s interaction with the earth & atmosphere

**HANDOUT’s OBJECTIVES:**
- familiarize student with EMR scattering basics in environment
- develop basic principles of radiative transfer in environment
- develop Lambert’s law & Schwarzschild equation, plus introduce remote-sensing applications of same

**Lord Rayleigh & sky blueness**

“Why is the sky blue?” is a good (and hardly trivial) starting point for studying EMR’s interactions with the earth’s atmosphere and oceans.

Water (whether vapor or liquid) is often invoked to explain sky blueness. After all, if ocean water is blue, shouldn’t that explain the sky’s color? Unfortunately, several meters of liquid water are required for water’s blue-green absorption minimum to be visible. If we condensed all atmospheric water vapor into a uniform shell around the earth, the shell would only be a few centimeters thick.

Visible scattering by water molecules differs little from that of other atmospheric gases. (At the molecular level, we distinguish between the scattering, or redirecting, of EMR and its absorption, or conversion to other wavelengths (or kinds) of energy.)

We retrace the ideas of 19th-century British scientist Lord Rayleigh, who showed that atmospheric molecules themselves are responsible for clear-sky color. Rayleigh reasoned that if a spherical particle’s radius $R_p < \ll$ than that of $\lambda$ light ($R_p/\lambda_{\text{light}} < 0.1$, the Rayleigh scattering criterion) its total scattering is $\propto$ its volume $V$. So when an EMR wave “illuminates” a molecule, scattering by its atoms is in phase because they are within $\lambda_{\text{light}}$ of each other.

Thus the amplitude of the scattered electric field $E_s \propto V$. Clearly $E_s$ also depends on the strength of the incident electric field $E_i$. Rayleigh knew that energy conservation required the intensity (aka irradiance) of the scattering to decrease with distance $r$ from the particle as $1/r^2$. In turn, this means that the amplitude $E_s$ decreases as $1/r$. What else describes $E_s/E_i$? Dimensional balance requires a variable with the units of length$^2$, and $\lambda^2$ is a plausible choice. So

$$\frac{E_s}{E_i} \propto \frac{V}{r \lambda^2} \quad \text{and} \quad \text{intensity} \left( \frac{E_s}{E_i} \right)^2 \propto \frac{V^2}{r^2 \lambda^4}.$$

Because each spherical particle’s $V = (4/3) \pi R_p^3$, relative scattering per particle is

$$\left( \frac{E_s}{E_i} \right)^2 \propto \frac{(R_p^3)^2}{r^2 \lambda^4} \propto \frac{R_p^6}{r^2 \lambda^4}.$$
Rayleigh’s arguments here are simply those of dimensional consistency and plausible (but not rigorous) use of EMR variables. Is it possible to get a similar answer from another tack?

Remember earlier that we said \( E_i = E_0 \sin(\omega t - \delta) \) describes the time and space dependence of an electric vector’s amplitude \((\omega = 2\pi \nu, \text{the angular frequency})\). We take the second time derivative of \( E_i \) to be a measure of the acceleration \( (E_s) \) that the EMR wave exerts on a charged particle (a scatterer). Thus:

\[
E_s \propto \frac{\partial^2 E_i}{\partial t^2} = -\omega^2 E_i = -\left( \frac{2\pi c}{\lambda} \right)^2 E_i,
\]

and the relative scattered intensity is given by:

\[
\left[ \frac{E_s}{E_i} \right]^2 \propto \left( \frac{2\pi c}{\lambda} \right)^4.
\]

Rayleigh’s relationship \( \left[ \frac{E_s}{E_i} \right]^2 \propto \frac{R_p^6}{\lambda^4} \) has two important consequences for remote sensing:

1) It describes the \( \lambda \)-dependence of visible scattering by molecules in the atmosphere as \( \propto \lambda^{-4} \), meaning that the atmosphere’s gaseous constituents scatter blue light (small \( \lambda \)) more efficiently than red light (larger \( \lambda \)). At the crudest level, this explains the blue sky.

2) It describes the raindrop size-dependence of radar scattering by rain. In other words, the reflected radiance of radar beamed at spherical raindrops is \( \propto R_p^6 \), meaning that \( N \) large raindrops will scatter much more than \( N \) smaller raindrops. Although there are fewer large raindrops than small drops per volume of rainy air, the \( R_p^6 \) factor usually means that the larger drops will return a stronger radar signal.

Note that we’ve jumped over an enormous range of particle sizes above. If an average atom’s nuclear radius is \( \sim 10^{-15} \text{ m} \) and a typical raindrop radius is \( 10^{-3} \text{ m} \), our examples of Rayleigh scatterers span 12 orders of magnitude.

In describing scattering, the ratio of particle size to wavelength is more important than absolute sizes. A measure of relative size is the size parameter \( x \):

\[
x = \frac{2\pi R_p}{\lambda} \quad \text{(Eq. 1)}.
\]
Rayleigh’s theory assumes that $\frac{R_p}{\lambda_{\text{light}}} < 0.1$. When this Rayleigh scattering criterion isn’t met, Rayleigh theory is inadequate and we must use the more general (and much more complicated) **Mie theory**. Below are Mie theory’s predictions about spectral scattering by a wide range of water-drop sizes. As a subset of Mie theory, Rayleigh theory holds only for the smallest drops (see bottom edge of nearest plane). Because of its power, Mie theory is used routinely in remote sensing applications.

**visible-\(\lambda\) scattering by 0.15–3.0 µm radius water drops**

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Rayleigh’s equation for scattering by atmospheric molecules says their scattering efficiency is $\propto \lambda^{-4}$. Since $\lambda_{\text{violet}} < \lambda_{\text{blue}}$, why isn’t the sky violet rather than blue? Our answer refers to human vision, but can easily be extended to satellite sensors.

1) Our visual systems are not equally sensitive to all wavelengths of light. In particular, note the low response at the violet end of the spectrum.

![Human visual sensitivity curve](image)

2) As we have seen earlier, the light source for skylight — sunlight — has less power in the short wavelength end of the visible spectrum.

3) Skylight does not consist of light of a single wavelength, but rather is a mixture of many $L_\lambda$. Our visual system converts this mixture of monochromatic radiances into three different signals, each averaged over a part of the visible spectrum.

For visible-$\lambda$ molecular (i.e., Rayleigh) scattering, $E_{\text{sct}}/E_{\text{inc}} \propto \lambda^{-4}$
4) We have neglected multiple scattering’s effect on sky color (i.e., repeated scattering of a photon before it reaches us). By describing scattering at the molecular level, we have tacitly implied that scattering by many molecules is the same as one molecule’s single scattering.

Multiple scattering is crucial to correctly interpreting both our ground-based views of the atmosphere and satellite images taken through it. In the atmosphere (or any other suitably dense medium), not only direct sunlight illuminates each molecule; so does sunlight scattered by other molecules. As shown below, an observer receives light scattered by molecules and particles all along a given line of sight.

![Diagram of light scattering](image)

Note that the pathlength in direction \(A\) > pathlength in direction \(B\). This simple fact explains a common observation: if molecular scattering \(\propto \lambda^{-4}\), why is the clear sky’s horizon white rather than blue?

The answer begins with the fact that atmospheric pathlengths increase as you look toward the horizon. What you see along paths \(A\) and \(B\) above is the sum of all scatterings along those paths.

Consider how this works to produce the white horizon:

1. Preferential blue scattering (\(\propto \lambda^{-4}\)) is identical at points \(A'\) and \(B'\).
2. But each subsequent scattering along \(A\) and \(B\) (i.e., closer to the eye) preferentially removes blue light first scattered at \(A'\) and \(B'\). The net result is that nearby scatterers contribute a larger amount of blue skylight to the total reaching your eye, while more distant scatterers contribute a smaller amount of reddened skylight.
3. Because \(A > B\), in direction \(A\) you see more scattered photons (i.e., brighter skylight) that are less dominated by blue (i.e., slightly reddened skylight).
4. For the longest pathlengths (at the horizon), this produces a brighter, whiter sky.
To analyze the multiple-scattering problem of horizon whiteness quantitatively, we begin by distinguishing between single- and multiple-scattering media. Our analysis will also tell us how satellites are affected by multiple scattering in the atmosphere.

We assume that all scatterers and absorbers in an idealized homogeneous medium are identical spheres of radius $R_p$ that are uniformly distributed. Consider a slab of thickness $h$ and area $A$ that’s lit by light of intensity $I_0$. The number of scatterers per unit volume in the slab is the particle number density $N$.

Every photon encounter with a particle in the slab results either in absorption or scattering (which together we call **extinction**), and except when the photon is scattered exactly in the direction it was originally headed (the forward direction), every encounter causes a reduction in $I_0$.

Over area $A$ and infinitesimal thickness $dh$ in the slab there are $NA$ particles ($i.e.$, \{number/volume\}*area*thickness = \{number/volume\}*volume = number of particles).

Now we **claim** (rather than prove) that each particle’s ability to scatter and absorb EMR is $\propto$ its projected cross-sectional area, $\pi R_p^2$. $C_{\text{ext}}$ denotes a particle’s **cross-sectional area for extinction** (for large particles, $C_{\text{ext}} = 2\pi R_p^2$). So $dh$’s total “target area” of scatterers is $NA C_{\text{ext}} dh$ (= \{# of particles\}*{area per particle\}).

As photons traverse $dh$, their numbers ($I_0$) are reduced by the differential amount $-dI$, and the number of photons intercepted per unit area is:

$$-dI = \frac{I N A C_{\text{ext}} dh}{A} = I N C_{\text{ext}} dh,$$

or,

$$\frac{dI}{I} = -N C_{\text{ext}} dh = -\beta_{\text{ext}} dh \quad \text{(Eq. 2)},$$

where $\beta_{\text{ext}} \equiv N C_{\text{ext}}$ defines the (volumetric) **extinction coefficient** $\beta_{\text{ext}}$.

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\( \beta_{\text{ext}} \) has the units of \( \text{length}^{-1} \) (\( \beta_{\text{ext}} = N \, C_{\text{ext}} \) is an area/volume). \( \beta_{\text{ext}} \) can be used to calculate the probability \( p \) of a photon being extinguished \( (i.e., \) either scattered or absorbed) in some thickness \( h \) of the medium \( \{ p = 1 \cdot \exp(-\beta_{\text{ext}} h) \} \). Put another way, \( \frac{1}{\beta_{\text{ext}}} \left( \text{units} = \frac{1}{\text{length}^{-1}} \right) \) is the average distance that a photon travels in the medium before being extinguished (the \textbf{photon mean free path}). We can break \( \beta_{\text{ext}} \) into separate scattering and absorption coefficients with \( \beta_{\text{ext}} = \beta_{\text{abs}} + \beta_{\text{sct}} \).

Eq. 2 \( \left( \frac{dI}{I} = -\beta_{\text{ext}} \, dh \right) \) is a differential equation whose general solution is:

\[
I = I_0 \exp(-\beta_{\text{ext}} h) \quad \text{(Eq. 3)},
\]
a result that’s called \textbf{Lambert’s law} of exponential attenuation.

Lambert’s law strictly applies to losses from a perfectly collimated \( (i.e., \) parallel) beam in \textit{exactly} the forward direction. It assumes that \textit{all} photons are forever lost from sight once they are scattered or absorbed. In other words, Lambert’s law is implicitly a single-scattering model.

What problems does this particular single-scattering model of the atmosphere pose?

First, realistic values for \( \beta_{\text{ext}} \) (or \( \beta_{\text{abs}} \) and \( \beta_{\text{sct}} \)) produce unrealistically large rates of beam extinction. For example, on a clear night the atmosphere typically has \( \beta_{\text{sct}} \sim 0.2 \text{ km}^{-1} \) (\( \beta_{\text{abs}} \rightarrow 0 \) here). If we uncritically apply Lambert’s law, we claim that the intensity (aka radiance) of a bright light seen 20 km away is attenuated by a factor of \( \frac{I}{I_0} = \exp(-0.2 \text{ km}^{-1} \times 20 \text{ km}) \sim 0.018 \), or to less than 2% of its original radiance. Does this square with common experience?

Second, Lambert’s law claims that the sun’s visible radiance will be \( \sim 5 \) times greater above our atmosphere than at its bottom \( (i.e., \) \( I/I_0 \sim 0.19 \)). In fact, attenuation in the visible is much less, with a typical measured \( I/I_0 \sim 0.67 \).

Third, Lambert’s law says that the daytime clear sky will be black, because we can see only directly transmitted solar photons. All photons removed from the direct solar beam are irretrievably lost. (Actually, most skylight \textit{is} due to singly-scattered solar photons, but the scattering is at angles other than 0° from the direct beam.)

Lambert’s law is not erroneous; we’ve simply applied it inappropriately. However, we \textit{can} use it in a single-scattering medium, provided that we ignore the law’s lack of directional scattering. We now define a single-scattering medium quantitatively.

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If a medium’s physical thickness $h \ll \text{photon mean free path } \text{(i.e., } h \ll \frac{1}{N C_{\text{ext}}})$, then by definition, photons are unlikely to be scattered even once, let alone a second time.

The inequality $h \ll \frac{1}{N C_{\text{ext}}}$ defines a medium in which multiple scattering is extremely unlikely, so we call it a single-scattering medium. Take “$\ll$” to mean “$< 0.1$” here, so that the single-scattering criterion can be rewritten as:

$$\text{single scattering iff } h N C_{\text{ext}} < 0.1 \quad \text{(Eq. 4)}.$$

Note that our definition depends on both the particle (or molecule) number density $N$ and on the particles’ sizes $C_{\text{ext}}$. If any of $h$, $N$, or $C_{\text{ext}}$ increase in a real material, it is less likely to be a single-scattering medium.

Let’s apply Eq. 4 to two common atmospheric media that often vex satellite oceanography: the clear atmosphere and clouds. Do either of these act as multiple-scattering media?

1) the clear atmosphere: Assume that the atmosphere can be represented as an 8-km layer in which density is constant and has the value measured at the earth’s surface. Because visible absorption is nil ($\beta_{\text{abs}} \to 0$), take $\beta_{\text{sct}} \sim N C_{\text{ext}} = 0.2 \text{ km}^{-1}$.

Here $h N C_{\text{ext}} = (8 \text{ km} \times 0.2 \text{ km}^{-1}) = 1.6$. Clearly nadir (downward looking) visibility could not quite be modeled accurately if multiple scattering is ignored. However, given all the approximations that we’ve used, the 1.6 value most likely means that this is a borderline case.

Obviously the atmosphere departs more from single scattering as we look toward the horizon, whether on the ground or from space. In this case, taking $h > 20 \text{ km}$ increases $h N C_{\text{ext}}$ by at least a factor of 2.5. In fact, data-analysis algorithms for limb-viewing satellites have to assume that multiple scattering prevails.

2) a cumulus cloud: $N \sim 300 \text{ cm}^{-3}$ and $R_{\text{p}} \sim 10 \mu\text{m} = 10^{-3} \text{ cm}$, making $C_{\text{ext}} = 2\pi R_{\text{p}}^2 \sim 6.28 \times 10^{-6} \text{ cm}^2$ (for cloud droplets, again $\beta_{\text{abs}} \to 0$). A minimal cumulus thickness is $h = 100 \text{ m} = 10^4 \text{ cm}$.

Now $h N C_{\text{ext}} = (10^4 \text{ cm} \times 300 \text{ cm}^{-3} \times 6.28 \times 10^{-6} \text{ cm}^2) \sim 18.85$. So cumulus clouds are not single-scattering media. In fact, multiple scattering from persistent cloud cover over the oceans is a major hindrance to satellite remote sensing.
Physical vs. optical pathlengths

As our discussions of molecular scattering and Lambert’s law indicated, an object’s absolute physical dimensions are not important for EMR propagation. Rather, it is size (or depth) scaled by the EMR itself that determine the nature of scattering and extinction.

For example, Rayleigh scattering depended on the ratio of \( \frac{R_p}{\lambda_{\text{light}}} \), and the potential for multiple scattering depended on the photon mean free path.

We now define the optical pathlength \( \tau \) as:

\[
\tau = \int_0^s \beta(\lambda,s,\theta) \, ds \quad \text{(Eq. 5)}.
\]

Note that \( \beta \) may be an extinction, scattering, or absorption coefficient, and that it can vary with: 1) wavelength \( \lambda \), 2) physical distance \( s \), 3) view direction \( \theta \). Physically, \( \tau \) is a medium’s physical thickness scaled by its photon mean free paths.

In developing a multiple-scattering criterion, Eq. 4 used a simplified version of \( \tau \) (\( \tau = h \, N \, C_{\text{ext}} \), with \( s = h \) and \( \beta_{\text{ext}} = N \, C_{\text{ext}} \)). There we assumed that the medium was homogeneous, but obviously real media won’t be homogeneous in general.

Can we describe scattering and emission in inhomogeneous media? Yes, but not as easily as when we developed Lambert’s law. Remember there we neglected: 1) the spectral and spatial variability of \( \tau \), 2) the effects of multiple scattering.

We start by considering a differential (spherical) volume of air \( dV \), although any medium will do. In general, this sphere of diameter \( ds \) receives radiances \( L(\lambda,s,\omega(\theta)) \) from \( 4\pi \) steradians and then both emits and scatters these \( L \) into \( 4\pi \) steradians.
Because $\tau_\lambda = \int_0^s \beta_\lambda ds$, we know that $d\tau_\lambda = (\beta_{\lambda,\text{sc}})ds$ if we neglect the scattering medium’s absorption. This approximation is perfectly acceptable in the atmosphere in the visible, and so $\beta_{\lambda,\text{sc}} \approx \beta_{\lambda,\text{ext}}$.

Lambert’s law \( \left( \frac{dL}{L} = -\beta_{\text{ext}} dh \right) \) can be restated in its differential form as:

\[
\frac{dL_\lambda}{ds} = -\beta_\lambda L_\lambda,
\]

where, for light in the atmosphere, we temporarily drop the distinction between extinction and scattering.

As before, we are saying that differential transmission losses in $L_{\lambda,\text{out}}$ (the radiances leaving $dV$) are $\propto$ the negative of some unscattered $L_{\lambda,\text{in}}$ (incoming radiances) times the scattering coefficient. This is just Lambert’s law restated.

However, $dV$ is also a source of $L_{\lambda,\text{out}}$ that originates as scattered $L_{\lambda,\text{in}}$ coming from many different $\omega(\theta)$. Collectively, we define these differential scattered radiances contributing to $L_{\lambda,\text{out}}$ as $\frac{dL_\lambda}{ds} = j_\lambda$, a source function. Because $j_\lambda$ is a differential contribution to $L_{\lambda,\text{out}}$ in $dV$, we can easily change Lambert’s law to:

\[
\frac{dL_\lambda}{ds} = j_\lambda - \beta_\lambda L_\lambda.
\]

Now we have considered both differential losses and gains to the total $L_{\lambda,\text{out}}$ heading toward our eyes. We will skip over the details of defining $j_\lambda$ mathematically. However, note that it rarely is simple to describe analytically because it depends on variations in both $\beta_\lambda$ and $L_{\lambda,\text{in}}$ as functions of direction $\theta$. For skylight, $j_\lambda$ includes the spectrum of direct sunlight after its attenuation by atmospheric scattering.

We can restate our modified equation in terms of $\tau_\lambda$, which has the advantage of making all terms spectral radiances and making our solution independent of physical distance per se. Then we have:

\[
\frac{dL_\lambda}{\beta_\lambda ds} = \frac{dL_\lambda}{d\tau_\lambda} = J_\lambda - L_\lambda, \quad \text{(Eq. 6)}
\]
where \( J_\lambda = \frac{j_\lambda}{\beta_\lambda} \) is still a source function, this time scaled by the photon mean free path.

Eq. 6 \( \left( \frac{dL_\lambda}{d\tau_\lambda} = J_\lambda - L_\lambda \right) \) is called the Schwarszchild equation, or the equation of radiative transfer (acronym = RTE). It describes fairly completely how the spatial variability of scatterers and the angular variability of \( L_{\lambda,\text{in}} \) affect the light (or other EMR) that ultimately reaches us.

Note that while we ignored absorption above, Eq. 6 is general enough that we can easily include it. As it stands, Eq. 6 is a first-order linear differential equation. However, both we and satellite sensors see the macroscopic integral results of these microscopic differential processes. So we must integrate the Schwarszchild equation to put it to practical use.

First, multiply Eq. 6 by the integrating factor \( e^{\tau} \) to get \( \frac{e^{\tau} dL}{d\tau} = J e^{\tau} - L e^{\tau} \)

which we rearrange as \( e^{\tau} dL + L e^{\tau} d\tau = J e^{\tau} d\tau \) (Eq. A). The rules of calculus tell us that \( d(e^{\tau} L) = e^{\tau} dL + L e^{\tau} d\tau \) (Eq. B). For clarity, we temporarily drop the \( \lambda \)-subscript, remembering that in general both \( J \) and \( L \) are \( f(\lambda, \tau, \omega(\theta)) \).

Substituting (B)’s left side for (A)’s left side yields \( d(e^{\tau} L) = J e^{\tau} d\tau \), which can be integrated from \( \tau = 0 \) to \( \tau = \tau_f \), where \( \tau_f \) is the final optical depth in which we are interested. Then we get \( (e^{\tau_f})L_f - L_0 = \int_0^{\tau_f} (J e^{\tau}) d\tau \), which can be solved for \( L_f \), the net radiance at \( \tau_f \):

\[
L_f = L_0 e^{-\tau_f} + \int_0^{\tau_f} (J e^{-(\tau_f - \tau)}) d\tau
\]

(Eq. 7).

Term \( A \) is the attenuated source radiance, term \( B \) is an “airlight” term representing the cumulative effects of scattering by each elemental volume \( dV \) of the atmosphere along our line of sight from \( \tau = 0 \) to \( \tau_f \).

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So at visible $\lambda$ in the atmosphere, term $A$ tallies source radiances that are *not lost* due to scattering over $\tau_f$ (aka Lambert’s law) and term $B$ tallies *gains* of scattered radiance from the intervening air. In the oceans, $\tau_f$ also includes absorption.

Note that in the visible the source function $J_\lambda$ is due almost entirely to scattering. However, in general, both scattering and emission contribute to $J_\lambda$. In the atmosphere, emission of EMR dominates Eq. 7 in the infrared.

The Schwarzschild equation (Eq. 7) is not simple to evaluate because real atmospheric $J_\lambda(\tau)$ do not yield airlight integrals (term $B$) that can be evaluated analytically. Thus Eq. 7 usually must be integrated numerically.

However, what form does Eq. 7 take if we assume a simple (and fairly realistic) behavior for $J$ along horizontal paths in the atmosphere, namely that $J \neq f(\tau)$? In other words, we assume that sunlight’s spectrum does not change anywhere along a horizontal path through the atmosphere. (Why is this not quite true?) If $J \neq f(\tau)$

{assumption #1}, then we have

$$L_f = L_0 e^{-\tau_f} + J \int_0^{\tau_f} (e^{-f(\tau_f - \tau)}) d\tau.$$

If $\tau_f \to \infty$ {assumption #2}, Eq. 7 becomes $L_f = O + J(1 - O) = J$. So for an infinitely long optical path, *only* airlight is visible in an object’s direction, not the object itself (its intrinsic radiance, $L_0$, is completely extinguished by the intervening air). For example, on hazy days distant mountains “disappear” into the intervening airlight.
Let's now move our vantage point to that of a satellite attempting to measure sea-surface temperatures (SST). Here in the infrared, $\beta_{\text{ext}} \sim \beta_{\text{abs}}$ because $\beta_{\text{sct}} \to 0$. Because the atmosphere radiates as a blackbody in the IR, not only will clouds contribute to the signal received at the satellite, so will the atmosphere itself.

Now we ask: can we isolate the temperature contributions that each vertical section of the atmosphere makes to the total received signal (and thereby compensate for the entire atmosphere in deriving SST)?

Now we drop our restriction that the atmosphere has constant density and pressure with height. In an isothermal atmosphere (i.e., $T \neq f(z)$), the pressure $p(z)$ at some altitude $z$ is given by $p(z) = p_0 \exp(-z/H)$. $H$ is the atmosphere's scale height, the 8-km thickness that we used earlier, and $p_0$ is the surface pressure.

Starting with this assumption, we can rewrite the Schwarzschild equation as:

$$L_f = L_0 e^{-\tau_f} + \frac{J}{H} \int_{z(0)}^{z(\tau_f)} \left[ \exp(-\tau_f p/p_0) \tau_f p/p_0 \right] dz ,$$

where the term $A$ in $[\ ]$ is now the contribution per $dz$ to the radiance received at the satellite. Term $A$ is of the form $x e^{-x}$, which has a maximum at $x = 1$. In this problem, $x = \tau_f p/p_0$.

What does this mean physically? At some pressure level $p(z)$ in the atmosphere, there is a maximum contribution to the atmospheric IR emission received by the satellite, $L_{\text{sat}}$. The higher in the atmosphere this level is, the less levels below $p(z)$ (and the sea surface itself) contribute to $L_{\text{sat}}$.
Because the factor $\tau_f p/p_0$ depends on $\beta_{\text{abs}}$ as well as $p$, the more highly absorbing the atmosphere is, the harder it is to “see” clearly to the surface in the IR. In other terms, the more opaque the atmosphere, the more difficult is reliable satellite remote sensing of SST.

Schwarzschild (or radiative transfer) equation integrand as $f(p, \tau)$ in an isothermal atmosphere