What are "all the colors of the rainbow"?

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Both folklore and theory imply that naturally occurring rainbows display a wide range of nearly pure colors. However, digital image analysis of color slides shows that the natural rainbow's colors are not especially pure and that the bow's background causes much of this desaturation.

I. Introduction

For most of us, the phrase "all the colors of the rainbow" evokes an image of the natural rainbow[1] as a paragon of color variety and vividness. Indeed, both our language and art often invoke the rainbow as a color palette without equal.[2] Yet as a color standard, the rainbow has an oddly contentious history. For example, arguments about the number of rainbow colors date to antiquity, with observers as keen as Aristotle[3] (who favored three colors) and Seneca the Younger[4] (who favored an indefinite number) among the disputants. That this disagreement still persisted in Georgian England[2] (and indeed to the present) hints that the rainbow poses special perceptual problems.

Colorimetry is ill-suited to answering epistemologically thorny questions about the number of rainbow colors. In addition, colorimetry does not describe how simultaneous color contrast, color constancy, and memory color influence our perception of the rainbow. However, colorimetry can begin to address the important question of what are "all the colors of the rainbow". Researchers have asked this question about rainbow theories applied to a single raindrop,[5–10] but no one has asked it quantitatively about the natural rainbow. In fact, recent rainbow theory has largely ignored human trichromatic processing of rainbow light, treating it as a superfluous adjunct to the detailed spectra of theory and the laboratory. However, our recently developed technique[11] lets us colorimetrically analyze photographs, thus opening up a whole new range of quantitative color information on the rainbow. These colorimetric data have the potential to tell us which rainbow theories adequately account for the natural bow.

II. The Rainbow in Context: Influences on the Natural Rainbow's Appearance

Some 50 years ago, Humphreys pointedly noted that "the 'explanations' generally given of the rainbow [in textbooks] may well be said to explain beautifully that which does not occur, and to leave unexplained that which does."[12] Many nontextbook factors that affect the color and luminance of the natural rainbow still go largely unconsidered. These factors include: (1) the angular divergence and coherence of sunlight, (2) the optical path length of rain showers, (3) the spectrum of raindrop sizes, (4) aerosol scattering and absorption, (5) aerodynamic distortion of raindrops, and (6) illumination of the rainbow's background.[13–18] To do full justice to the natural rainbow, we must do much more than uncritically apply a rainbow theory to monodisperse, spherical raindrops that are illuminated by perfectly collimated light and seen against a black background.

All the factors listed above can be incorporated, with varying degrees of difficulty, into various rainbow models. However, rather than immediately launching into a lengthy modeling exercise, we ask an important preliminary question in this paper: how well can we separate the rainbow's intrinsic colors[19] from those of its background? Knowing the answer to this question clearly affects what we regard as important in modeling the natural rainbow's intrinsic colors. At the same time, we want to know what range of observed colors we might expect in the natural rainbow.

Tricker notes that the "colours of all [natural] rainbows are dilute and contain much white light," yet his color analysis of Airy theory shows high purities at various deviation angles.[9] This unintended discrepancy arises because Tricker, like others,[6–8,10] illustrates rainbow color theory by analyzing single-droplet rainbows *sans* background. Such an approach is perfectly

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acceptable, provided that we remember where it diverges from nature.

In reality, the rainbow's intrinsic colors are mixed additively with those of its background, and the bow's observed colors depend on the background's relative luminance. For example, while we can demonstrate that the sky inside the primary bow should be brighter than that outside, sometimes this distinction may be difficult to observe (Plates 1 and 2). Several factors can cause this: a rainshaft of limited depth, a background whose luminance is not uniform, or sunlight attenuated by haze. In each case, the low intrinsic luminance of the primary bow compared with its background means that only the rainbow's pattern of colors, not its inherent brightness, will be noticeable. The colors of the secondary and supernumerary bows are often imperceptible because their contrast with the background is even lower than that of the primary. For natural rainbows of any order, low intrinsic luminance yields pastel colors.

III. Quantifying the Natural Rainbow's Observed Colors

Plates 1–5 illustrate a wide variety of rainbows, ranging from the pedestrian to the spectacular. To quantify just how pedestrian or spectacular these are, we begin by electronically digitizing the original color slide for each figure, along with slides of a card containing many different color samples. We use the same kind of color film to photograph the rainbow and the color card. The time and location of each plate specify the sun's elevation, which in turn lets us estimate its spectrum. If the same sunlight spectrum generates the rainbow and illuminates the color card, we can use the card's known colors to describe the rainbow's unknown colors quantitatively.[11] Note that small errors in estimating the spectrum of direct sunlight are not a crucial problem here; we are primarily interested in chromaticity range or gamut, rather than in absolute chromaticities.

Our metric of color is the CIE 1976 UCS (uniform-chromaticity-scale) diagram, a plane of uniform luminance that is approximately isotropic for color differences.[20] Because the UCS diagram is a linear transformation of the 1931 CIE spectrum locus, it is not truly perceptually isotropic. However, the UCS diagram's near-isotropy means that equal Cartesian distances within it nearly correspond to equal color differences. This property will be useful as we compare the color gamuts of different rainbows. To convey a sense of the 1976 UCS chromaticities, we have calculated and displayed the colors that fill the diagram's interior (Plate 6). However, remember that, for purely additive color mixing, we can reproduce all perceptible colors only if we use an enormous number of monochromatic lights. Our version of the UCS diagram was photographed from a computer's red-green-blue color monitor. Obviously trichromatic systems like color TV and film can only approximate certain parts of the UCS diagram (to mention nothing of the color shifts introduced by photography and printing). Thus chromaticity diagram illustrations are only qualitatively useful guides to CIE color space.

Given digitized versions of Plates 1-5 and their corresponding colorimetric calibrations, we can now analyze the figures' colors. Fraser[16–18] and Gedzelman[14,15] have described how rainbow color and brightness vary with clock angle.[21] However, most observers define the gamut of rainbow colors as that seen when they look *along* radii of the bow. Thus in our initial analysis, we examine changes in chromaticity and luminance along rainbow radii. Plates 1–5 were photographed with several 35-mm cameras and with lenses of different focal lengths. In addition, we cropped each photograph somewhat differently during digitizing. As a result, rainbow radii are not readily comparable among the different digitized images. Because of this and because we are more interested in overall color gamut than in chromaticity as a function of rainbow radius, our radial luminance scans [Figs. 1(b)–5(b)] simply indicate the direction of the antisolar point. The ordinate in Figs. 1(b)–5(b) is luminance relative to that of a diffuse white (i.e., spectrally nonselective) surface whose reflectance is 100%.

Within the digitized images, we encounter two kinds of color change at a given rainbow radius. First, both color and luminance change for the reasons that Gedzelman and Fraser describe.[14–18] Second, errors in quantizing the video camera's analog signal mean that our digitized images are slightly noisy. Before analyzing the images colorimetrically, we reduce this noise with several smoothing techniques. However, some random variations in pixel values still exist. Further smoothing is desirable, especially when we subtract the rainbow's background luminances. Thus at a given rainbow radius in a digitized image, we average chromaticities over many clock angles. Thus we are smoothing not only noise, but also some of the real, systematic color variations described in Refs. 14–18. In fact, however, chromaticity gamuts showed little systematic change, even when we averaged over a 40° range of clock angles.

Figures 1(a)–5(a) show the chromaticity curves for radial scans across the rainbows of Plates 1–5. In Figs. 1(a)–5(a), the thick lines are the chromaticities of rainbows as seen by naked-eye observers (i.e., with background luminances included), and the chromaticities of direct sunlight are marked with an ×. It is difficult not to be startled at the smallness of the rainbows' color gamuts, especially those of the paler bows [Plates 1 and 2 and Figs. 1(a) and 2(a)]. Colorimetrically speaking, the rainbows of Plates 1 and 2 scarcely exist. However, we photographed these bows because they seemed vivid, not because they struck us as especially pallid.

How can we quantify the color gamuts of these and other rainbows? While recognizing that the range of colors in most natural scenes defies easy classification, we nonetheless introduce an approximate measure of this range, the normalized colorimetric ganut $\hat{g}$. First we determine a chromaticity curve's unnormalized colorimetric gamut g. We start by finding the curve's

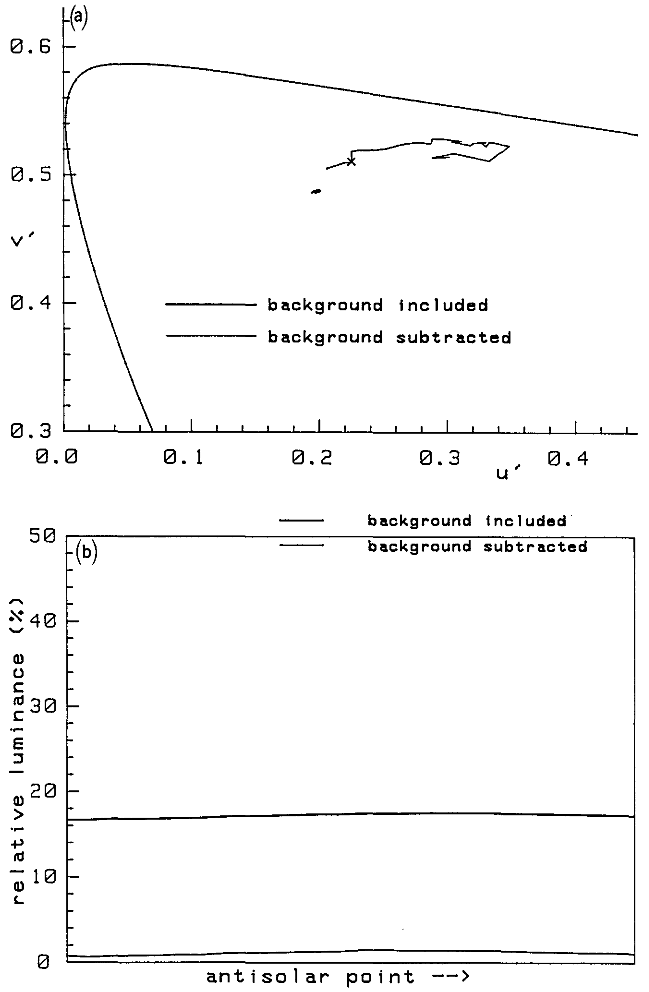

Fig. 1. (a) Portion of the CIE 1976 UCS diagram (spectrum locus drawn with dot–dash line) showing the colorimetric gamut $\hat{g}$ of the Fig. 1 rainbow, both with background luminances included ($\hat{g}$ = 0.00701, thick line) and subtracted ($\hat{g}$ = 0.134, thin line). (b) Radial variation of relative luminances in the Plate 1 rainbow, both with background luminances included (thick line) and subtracted (thin line). Each point on the curves is an average over many rainbow clock angles.

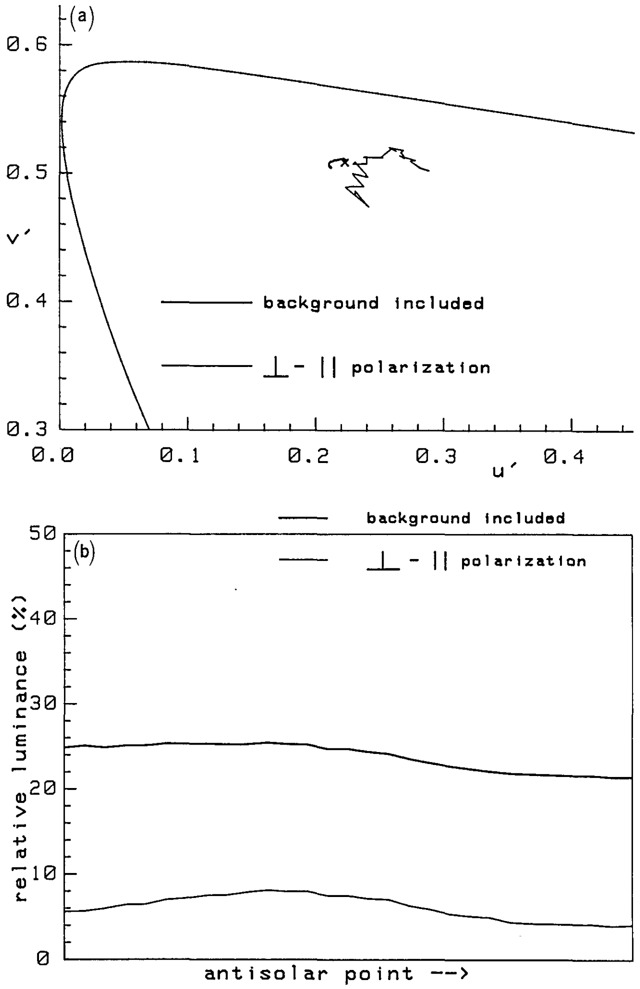

Fig. 2. (a) Colorimetric gamut of the Plate 2 rainbow, both with background luminances included ($\hat{g}$ = 0.0195, thick line) and for the ‖-polarized component colorimetrically subtracted from the ⊥ component ($\hat{g}$ = 0.0746, thin line). (b) Radial variation of relative luminances in the Plate 2 rainbow, both with background luminances included (thick line) and for the difference of the ⊥ and ‖ polarization components (thin line).

average chromaticity (here, the mean u', v'), and then we calculate the root mean square (rms) Cartesian distance of the curve's chromaticities from its $\overline{u'}$, $\overline{v'}$. Thus for a chromaticity curve of N points,

$$g = \left(\left\{ \sum_{i=1}^{N} [(u_i' - u')^2 + (v_i' - v')^2] \right\} \Big/ N \right)^{1/2}. \qquad (1)$$

Like any other chromaticity curve, the spectrum locus also has a colorimetric gamut, g_s. Using the spectrum locus as our color paragon, we use its gamut to normalize any other chromaticity curve's gamut g such that

$$\hat{g} = g/g_s. \qquad (2)$$

Thus $\hat{g}$ ranges from 0 to 1, independent of the colorimetric system used ($\hat{g} \equiv 1$ for the spectrum locus). However, the greater a color space's perceptual anisotropy, the less $\hat{g}$ will correspond to our impression of color gamut. We use $\hat{g}$ rather than an analogous measure, mean colorimetric purity, for two reasons. First, as shown below, background color predominates in pastel rainbows, and mean purity would then chiefly depend on the background's average purity rather than the rainbow's. Second, the color of sunlight changes with solar elevation, meaning that the achromatic point changes too. For our purposes, it seems unnecessarily confusing to try to associate color purity with so mobile a reference point.

We may not naively believe that $\hat{g} = 1$ for the natural rainbow (as the bow's folklore implies). In fact, Airy theory predicts that $\hat{g} = 0.405$ for a rainbow generated by a spherical droplet with a radius of 1 mm (the deviation angle ranges from 137° to 145°). For droplets with radii of 0.5 and 0.15 mm, Airy theory's $\hat{g}$ values are 0.378 and 0.372, respectively. However, when we include the natural rainbow's background in

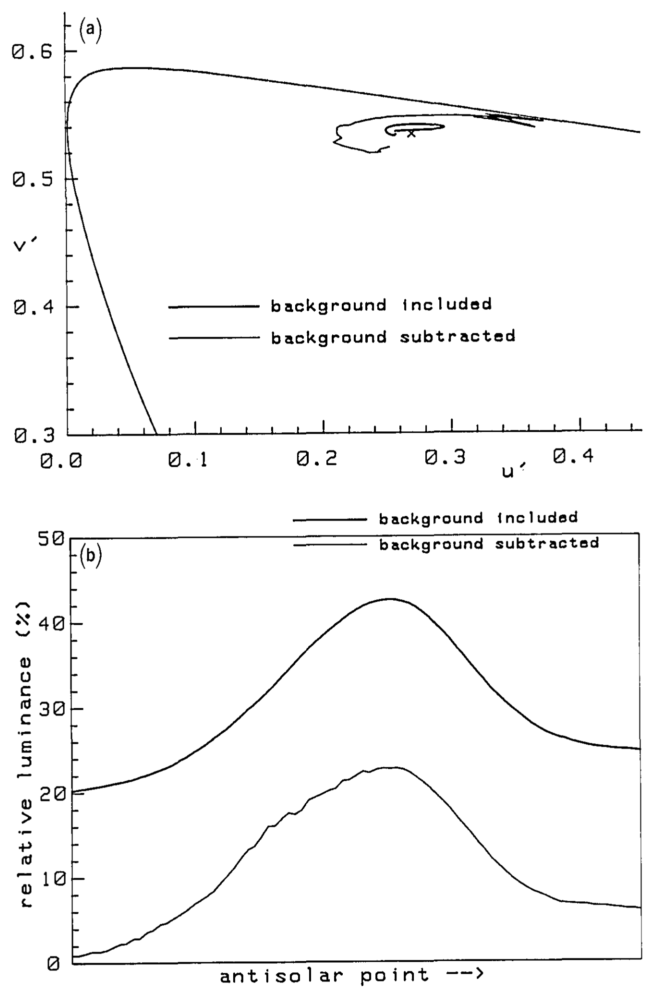

Fig. 3. (a) Colorimetric gamut of the Plate 3 rainbow, both with background luminances included ($\hat{g}$ = 0.0507, thick line) and subtracted $\hat{g}$ = 0.181, thin line). (b) Radial variation of relative luminances in the Plate 3 rainbow, both with background luminances included (thick line) and subtracted (thin line).

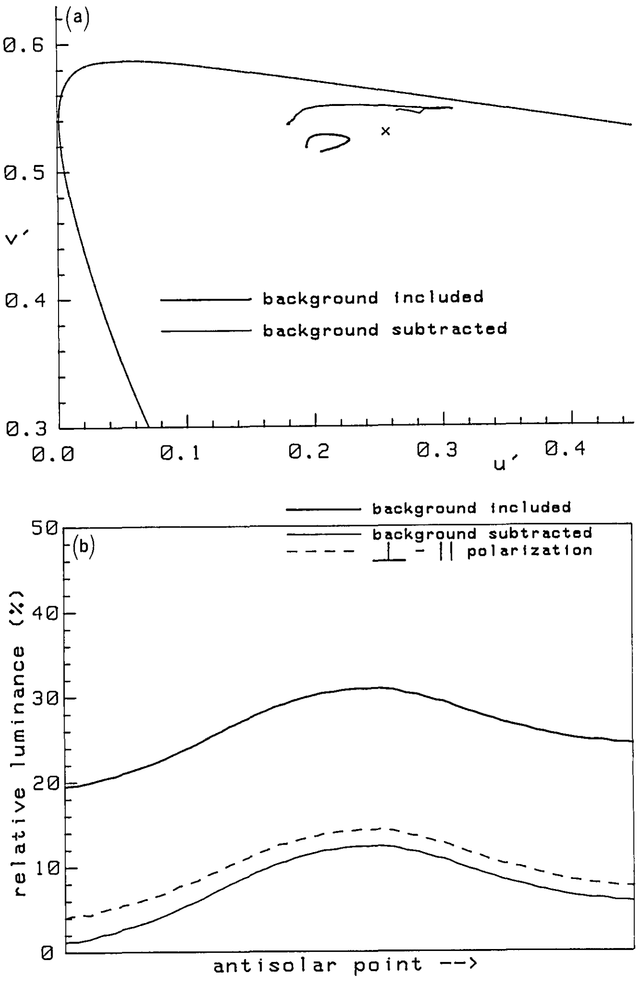

Fig. 4. (a) Colorimetric gamut of the Plate 4 rainbow, both with background luminances included ($\hat{g}$ = 0.0424, thick line) and subtracted ($\hat{g}$ = 0.156, thin line). (b) Radial variation of relative luminances in the Plate 4 rainbow, both with background luminances included (thick line) and subtracted (thin line). The dashed curve is the luminance difference of the ⊥ and ∥ polarization components.

our measurements, $\hat{g}$ varies in Figs. 1 and 2 between the surprisingly small values of 0.00701 and 0.0195, respectively. Especially for Plate 1, we might be concerned that small errors in defining the rainbow's location within the digitized image will unduly compress the bow's average radial color gamut. Yet calculating $\hat{g}$ separately at each clock angle in Plate 1 and then averaging these numbers only increases $\hat{g}$ to 0.0136. Thus the rainbows of Plates 1 and 2, which were quite striking visually, nonetheless span <2% of the human color gamut. Because bows like these are not uncommon, the rainbow begins to seem a rather sorry color standard.

Even the spectacular rainbows of Plates 3–5 occupy much less colorimetric space than we might at first imagine. The rainbow in Plate 3 is the most colorful, with $\hat{g}$ = 0.0507. Plate 4 is a close second at $\hat{g}$ = 0.0424, and Plate 5 has the third-largest normalized color gamut of $\hat{g}$ = 0.0342 (all these figures include the background). We would be hard pressed to find rainbows more vivid than these, and the best of them spans a colorimetric gamut four to seven times greater than that of the Plate 1 rainbow. Yet none of our rainbows has a color gamut that is even 6% of the spectrum locus. Is the rainbow's reputation for nonpareil colors undeserved? We leave this poser for now, and turn to the related question of the background's influence on rainbow colors.

IV. Isolating the Natural Rainbow's Intrinsic Colors

We use two techniques in trying to separate the rainbow's intrinsic colors from those of its background. Both techniques involve colorimetrically subtracting an estimate of background color from that of the rainbow at each pixel within our digitized images. The first method has several steps: (1) at each clock angle we measure the chromaticities and luminances of the 20 pixels that lie just outside the primary rainbow in Alexander's dark band, (2) we then average these chromaticities and luminances, and (3) we colorimetrically

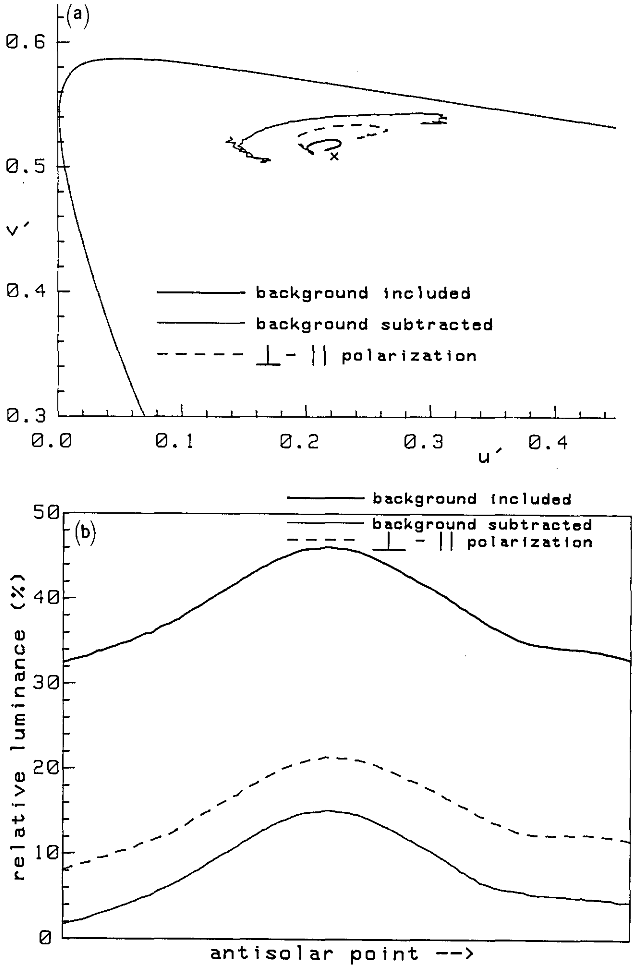

Fig. 5. (a) Colorimetric gamut of the Plate 5 rainbow, both with background luminances included ($\hat{g}$ = 0.0342, thick line) and subtracted ($\hat{g}$ = 0.203, thin line). The dashed chromaticity curve is the colorimetric difference of the ⊥ and ‖ polarization components; its $\hat{g}$ = 0.0883. (b) Radial variation of relative luminances in the Fig. 5 rainbow, both with background luminances included (thick line) and subtracted (thin line). The dashed curve is the luminance difference of the ⊥ and ‖ polarization components.

subtract this color from each of the colors measured across the rainbow. Note that the average chromaticity and luminance change with clock angle, thus giving us an accurate local measure of background color. The result of the colorimetric subtraction is an estimate of the rainbow's intrinsic colors at the given clock angle. As noted above, the chromaticity curves plotted in Figs. 1(a)–5(a) show the radial variation of rainbow colors averaged over many clock angles. In each figure, follow the color sequence from the outside to the inside of the bow by tracing counterclockwise along the chromaticity curve.

The second method of isolating the rainbow's intrinsic colors exploits the fact that rainbow light is highly linearly polarized compared with light from the background.[9,22,23] We define the rainbow's perpendicular (⊥) polarization component as that seen through a linear polarizing filter when we align its transmission axis perpendicular to the scattering plane defined by the sun, a raindrop contributing to the bow, and the observer. At this polarizer orientation, the rainbow is at its brightest. However, if we rotate the polarizer 90°, the bow's much weaker parallel (‖) polarization component is essentially invisible. Light from the landscape and clouds is often largely unpolarized. Thus if we align a digitized image of a rainbow's ⊥-polarized component (Plate 5) with its ‖-polarized counterpart (Plate 7), we can colorimetrically subtract the two scenes pixel by pixel. Once again we have an estimate of the rainbow's intrinsic colors.

However, bear several important caveats in mind. We change the scattering plane's orientation as we look around the rainbow at various clock angles. Changing the scattering plane also means changing the orientation of the bow's ⊥-polarized component. Why then can we see the rainbow over many clock angles when we look through the polarizing filter? The answer to this question leads to our first caveat: the rainbow is not *completely* linearly polarized. This qualification explains why we can see rainbow light from many clock angles at a single filter orientation and why the rainbow does not vanish completely when we rotate the polarizer 90° from the position that yields the brightest rainbow. Thus when we speak of the rainbow's ⊥-polarized component, we mean an average ⊥-polarized component for a restricted range of clock angles. Our second caveat is that some background light sources (and indeed airlight itself) are partially polarized. Thus subtracting the ‖ component from the ⊥ component will sometimes yield brighter and less saturated rainbow colors than will subtracting the background directly. Finally, both colorimetric subtraction techniques will occasionally yield spurious results (i.e., negative luminances or purities >100%). We exclude all such perceptual impossibilities from our averages.

Figures 1(a)–5(a) show how gamuts are increased by both background and polarization subtraction (Plates 1 and 3 lack ‖-polarized counterparts). Not surprisingly, isolating the natural rainbow expands the gamut of its purities (but not necessarily its dominant wavelengths) considerably, and oranges of nearly 100% purity can occur. However, most other colors are much less saturated, and purities <10% are not uncommon. In Fig. 1(a) background subtraction increases $\hat{g}$ to 0.134, a 19-fold increase. Because the intrinsic rainbow's luminance is so small here [thin line, Fig. 1(b)], mixing its colors with the bluish cloud background makes the observed bow a sequence of blues modulated only slightly by red and yellow.

Note the jaggedness of the intrinsic bow's chromaticity curve in Fig. 1(a) (see Fig. 1). Although we plot only the average chromaticities for each rainbow radius, nonetheless our estimates of intrinsic colors will be noisier than those of observed colors. We quantify a chromaticity curve's rms noise n in a manner analogous to its gamut g. Each u', v' on a chromaticity curve is a local average of chromaticities from many different clock angles. The rms distance of each of those chromaticities from the locally averaged u', v' is n for that point on the curve. Thus at a chromaticity curve's ith point $\overline{u_i'}$, $\overline{v_i'}$, [see Eq. (1)], which is itself an average of chromaticities from M clock angles,

$$n_i = \left(\left\{\sum_{j=1}^{M} [(u_j' - u_i')^2 + (v_j' - v_i')^2]\right\}\Big/ M\right)^{1/2}. \tag{3}$$

Here we simply report n averaged over the entire curve. In Fig. 1(a) this n is 0.0663 for the intrinsic colors and 0.00367 for the observed colors, an 18-fold difference in noise. Thus we should regard the intrinsic chromaticity curve here with some skepticism.

In Fig. 2(a) (see Plate 2), polarization subtraction increases $\hat{g}$ from 0.0195 to 0.0746. Once again, attempting to isolate the intrinsic colors of a pastel rainbow increases colorimetric noise substantially; n increases from 0.00421 for the observed colors to 0.0459 for our estimate of the intrinsic colors, a factor of 11 increase. Figure 2(b) suggests a photometric reason why n increases less in Fig. 2(a) than in Fig. 1(a): the intrinsic rainbow luminances are substantially larger in Fig. 2(b).

In Fig. 3(a) the rainbow with the widest gamut of observed colors ($\hat{g}$ = 0.0507, see Plate 3) in our sample displays the second-largest gamut of intrinsic colors ($\hat{g}$ = 0.181). For the quite bright rainbow of Plate 3 [see Fig. 3(b)], subtracting the background increases n less than before. Specifically, n increases from 0.00697 to 0.0356 in going from observed to intrinsic colors. A characteristic feature of our chromaticity curves for intrinsic rainbow colors is that they are noisier (i.e., more jagged) at the bows' inner and outer radii than they are in their interiors. Clearly the smoother parts of the curves are associated with the higher luminances found within the rainbow (and thus with an improved signal-to-noise ratio).

Although we performed both background and polarization subtraction on the Plate 4 rainbow, we have drawn colorimetric results only for the former technique in Fig. 4(a). We do this because the two curves are essentially indistinguishable; $\hat{g}$ = 0.156 for background subtraction and $\hat{g}$ = 0.103 for polarization subtraction. The values for n are, respectively, 0.0149 and 0.0129. The close photometric agreement between the two subtraction techniques is evident in Fig. 4(b). For the observed colors, n = 0.00567.

The widest gamut of intrinsic rainbow colors is shown in Fig. 5(a) (see Plate 5). Here $\hat{g}$ = 0.203 for background subtraction but is more than halved to 0.0883 for polarization subtraction. The noise figures are also larger for background subtraction (n = 0.0282 versus n = 0.0101; n = 0.00325 for the observed colors). Oddly enough, the Plate 5 rainbow has the smallest observed chromaticity gamut ($\hat{g}$ = 0.0342) of our three vivid rainbows (Plates 3-5). Figure 5(b) indicates one possible reason for this disagreement about the rainbow colors in Plate 5. Note the large luminance difference between the two subtraction techniques. This difference suggests (although does not prove) that some of the discrepancies among our various techniques arise from relatively strong polarization of background and foreground light in Plate 7.

Thus, for Plate 5, polarization subtraction seems to have underestimated the gamut of intrinsic rainbow colors. However, we should not discard polarization summarily. First, if background light is not highly polarized, polarization subtraction can give intrinsic color estimates that are essentially indistinguishable from those obtained by background subtraction (see discussion for Plate 4). As our image analysis techniques evolve, we will be better able to distinguish between acceptable and unacceptable degrees of background polarization. Second, if background luminance changes markedly across the rainbow, polarization subtraction may well give more accurate results than background subtraction. To simplify matters in this paper, we have not analyzed rainbows with complex backgrounds.

V. Conclusions

Our techniques for isolating the natural rainbow's intrinsic colors yield chromaticities that approach those predicted by theory.[5–10] For a truly satisfactory fit between theory and observation, we still need to consider the aerodynamic and optical factors listed above. However, remember that we will almost never see natural rainbows' intrinsic colors (i.e., we do not see rainbows against black backgrounds). For even the brightest and purest of rainbows, the background desaturates the bow's intrinsic colors markedly. Our analysis of color gamut indicates that, at best, we may expect the bow's background to reduce $\hat{g}$ by a factor of 2, and a fourfold reduction is more typical. Obviously for pastel rainbows (e.g., Plates 1 and 2) the reduction in color gamut can be much larger.

Ultimately, we plan to compare the colorimetric analyses reported above with those measured by a fast-scanning spectroradiometer. Given the noisiness of some of our digital image data, spectroradiometry still seems a desirable alternative in analyzing rainbow colors. However, we do not need detailed *in situ* spectra to establish that the observed colors of naturally occurring rainbows are remarkably desaturated.

Returning to our earlier conundrum, do our colorimetric results mean that we must discard the old saw about the bow's colors? Not really. Although observed purities in even the most spectacular rainbows are well under 100%, we should remember how colorimetric purity is defined. Monochromatic light sources define the pure colors of the spectrum locus, and such lights are all but absent from the color environment in which we have evolved. While we *can* see light of 100% purity, we almost never *do* see it in nature.[24,25] Of all atmospheric colors, only the sun's reddened disk exceeds the rainbow's purity. By comparison, the blue sky has a theoretical maximum purity of only ~40%,[26] and observed sky purities will be even smaller. Among celestial displays, spectacular rainbows are unexcelled in presenting a wide range of reasonably pure hues. In essence, although "all the colors of the rainbow" span but a fraction of the human color gamut, they often give us a peerless natural color palette.

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