Although quantitative observations of rainbow spectra, colors, and luminances are needed for any comprehensive analysis of rainbow scattering theory, very little such data has been published. But new remote sensing tools now make possible the detailed spectral and colorimetric measurement of natural rainbows, which here are defined as bows seen in sunlit rain or water-drop sprays. To measure these often short-lived phenomena, both multispectral tools (colorimetrically calibrated RGB cameras) and hyperspectral tools (imaging spectrometers) are used to examine the spectral and angular fine structure of natural rainbows. Airy theory for aerodynamically flattened drops helps to explain some of these bows’ observed features, such as the reduced color gamuts caused by smaller drop sizes and low sun elevations $h_0$. However, other features such as the distinct blues seen in rainbows at higher $h_0$ are not well explained.

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**1. INTRODUCTION**

Published spectral and colorimetric measurements of natural rainbows and spraybows (i.e., bows seen in sunlit rain or sprays of water drops) are quite rare [1], including those made at high angular resolutions along rainbow radii [2,3]. Far more common are simulations of rainbow spectra or angular intensities scattered by isolated water drops or by optically thin polydisperse rain and cloud droplets [4–11]. A few researchers have also calculated how rainbow spectra and colors are influenced by (1) incident sunlight’s spectrum, (2) multiple scattering from optically thick water sprays or rainswaths [12–14], (3) sky or terrestrial background radiances [2,3], and (4) raindrop size and shape distributions [10,15].

Yet to my knowledge, no studies exist that quantitatively compare measured and modeled rainbow spectra using all four of these environmental factors. To fully describe rainbows and fogbows as seen in nature, factors (1)–(4) above must be added to models of rainbow scattering by drops that may be nonspherical [16–18]. This last requirement comes from the distortion of moving raindrops or spray drops due to aerodynamic forces [19], which here are assumed to produce flattening that is either symmetric (the Können model [16]) or asymmetric (the Sadeghi model [17]). Drop oblateness increases with drop size because larger drops undergo more aerodynamic flattening and, for tilted sprays, this flattening will not be along a vertical axis.

In this initial paper, I use the Können model (a modified Airy theory) and note that because Lorenz–Mie theory in its original form is limited to spherical particles, it is better suited to modeling cloudbows and fogbows rather than natural rainbows. Because the Können model can use either oblate spheroidal or purely spherical drops, I take advantage of this flexibility to see if the two drop shapes can cause any perceptible color differences. Finally, although rainbow scattering’s high degree of linear polarization [20–22] helps to separate rainbow spectra and colors from their often unpolarized backgrounds [2], measuring rainbow polarization *per se* is not my primary goal here.

**2. MULTISPECTRAL ANALYSIS OF RAINBOW CHROMATICITIES**

Figure 1 shows two basic observational parameters for natural rainbows. In it, the polar coordinates of rainbow radius $r$ and clock angle $\alpha$ are superimposed on a primary rainbow photographed at refracted sun elevation $h_0 = 13.9^\circ$. Radius $r$ increases from 0° at the antisolar point to ~42° at the primary bow’s red band, whereas $\alpha$ increases clockwise or counterclockwise from 0° at the rainbow’s vertical sides (not visible in Fig. 1) to 90° at its top (i.e., the 12 o’clock position). This symmetry about $\alpha = 90^\circ$ stems from the fact that at equal clock-angle distances from the antisolar meridian (where $\alpha = 90^\circ$), a left- or right-tilted cross section through an aerodynamically flattened raindrop will have the same shape.

Rainbow spectra are measured with an imaging spectrometer in Section 3 below, but Section 1 uses multispectral
ary bow, while the larger loop is that of the primary itself. The smaller loop corresponds to Fig. 1r rows point toward larger

balanced white

techniques developed earlier for three-channel red-green-blue (or RGB) cameras. With suitable colorimetric calibrations [23–25], most high-quality consumer digital cameras can provide accurate, angularly detailed colorimetric data from their RGB images. By having many more samples in my color target than before (177 vs. 24) [2], my latest calibrations yield more accurate colorimetric interpolations among sample colors. For example, Fig. 2 is based on RGB camera data from Fig. 1, and it shows part of the CIE 1976 uniform-chromaticity-scale (or UCS) diagram. On a point of terminology, observed spraybows or rainbows are measured bows that include background radiance, whereas measured intrinsic bows exclude background radiance (see Ref. [2] for further details).

Figure 2 includes such observed colorimetric features as a color-temperature interval along the Planckian locus and a local MacAdam color-matching ellipse whose semiaxes span one just-visible difference (or JND) [26]. Figure 2’s solid red curve plots the measured UCS $u'$, $v'$ chromaticities for Fig. 1’s primary rainbow as a function of radius $r$, and the arrows point toward larger $r$. The red chromaticity hook at small $r$ corresponds to Fig. 1’s nearly indistinguishable supernumerary bow, while the larger loop is that of the primary itself. Figure 2’s two other curves are the radial chromaticities $u'v'(r)$ predicted by the Können/Airy model [16] for size distributions of oblate spheroidal or spherical raindrops.

Figure 2’s model results include additive mixing of (a) Fig. 1’s mean measured $u'$, $v'$ cloud background in Alexander’s dark band outside the primary with (b) the intrinsic rainbow $u'$, $v'$ due to droplet scattering of a solar spectrum measured on another day at Fig. 1’s $h_0$. Here and in later simulations, rainwater’s real indices of refraction can vary with temperature, and their spectrum at visible wavelengths $\lambda$ is accurately fitted with a fourth-order polynomial. Figure 2’s simulations also include the effects of optically thin thunderstorm rain whose drop size distribution (or DSD) is calculated from the site’s small rainfall rate [27] measured concurrently by a nearby Doppler weather radar.

Even given these approximations, Fig. 2 is instructive. First, the Können/Airy model’s oblate spheroidal drops yield a radial chromaticity curve whose shape is more congruent with Fig. 2’s measured curve (e.g., note the similar tilt of the oblate-spheroidal supernumerary $u'v'(r)$). That neither model curve exactly matches (i.e., coincides with) the measured chromaticities is less important here because the illuminant spectrum is estimated, and a RGB camera’s absolute (as opposed to relative) reconstructed chromaticities depend on the accuracy of this estimate. Second, even given the same estimated illuminant, spherical and nonspherical raindrops produce perceptibly different rainbows (i.e., Fig. 2’s model curves often differ at a given $r$ by more than the local MacAdam JND).

In Fig. 3, a vivid primary rainbow is seen at a much lower refracted $h_0 = 2.84^\circ$. Compared with Fig. 2, this rainbow’s entire normalized color gamut $\hat{g}$ [28] shifts to lower color temperatures (i.e., the bow becomes redder [29]) in Fig. 4 and decreases in magnitude by ~15%. As in Fig. 2, the rain DSD in Fig. 4 is calculated from rainfall rates measured by a nearby weather radar. Switching raindrop shapes in Fig. 4’s Können/Airy model now has subtler consequences: the oblate spheroidal drops produce a $u'v'(r)$ chromaticity curve that (1) is slightly more congruent with the observed curve, (2) measurably reduces the model’s mean chromaticity error (i.e., moves the spheroidal curve closer to the observed chromaticities),
modeled rainbow chromaticities as white balance \( \sim \) the absolute Hyperspectral imaging largely eliminates errors in measuring BACKGROUNDS NATURAL RAINBOWS AND THEIR

3. HYPERSPECTRAL MEASUREMENT OF
ticity curve with respect to the Planckian locus.

CIE 1976 UCS diagram with curves of Fig.3

Fig. 3. Primary rainbow photographed at Dunkirk, Maryland on 30 September 2011 at refracted \( b_0 = 2.84^{\circ} \). iPhone 4 camera, shutter speed = \( 1/125 \) s, aperture = \( f/2.8 \), sensitivity = ISO80, and auto white balance \( \sim 3250 \) K.

Fig. 4. CIE 1976 UCS diagram with curves of Fig. 3’s observed and modeled rainbow chromaticities as \( f(\alpha) \). Mean observed \( u'v'(\alpha) \) are averaged over \( \alpha = 14.3^{\circ}–37.8^{\circ} \). Simulated chromaticity curves are from the Kön nen/Airy model [16].

and (3) produces a more realistic rotation of the model chromaticity curve with respect to the Planckian locus.

3. HYPERSPECTRAL MEASUREMENT OF NATURAL RAINBOWS AND THEIR BACKGROUNDS

Hyperspectral imaging largely eliminates errors in measuring the absolute chromaticities of rainbows and spraybows, as opposed to multispectral imaging’s accurate reconstruction of their relative chromaticities. Thus when properly calibrated, either imaging system can accurately measure the shape of a \( u'v'(\alpha) \) chromaticity curve, but a hyperspectral system will make smaller absolute chromaticity errors in measuring a given \( u', v' \) pair. This is because hyperspectral systems can sample the visible spectrum in scores of discrete, narrow spectral bands rather than the much wider bands of a three-channel RGB camera. Thus hyperspectral systems greatly reduce metamerism errors. For ephemeral natural rainbows, a multispectral system offers superior speed in setup and image capture, especially in low-light conditions such as those near sunset. This speed is a considerable advantage when we only need accurate relative chromaticities.

For this paper’s hyperspectral imaging, I use a Resonon, Inc. Pika II imaging spectrometer [30]. It consists of a digital camera that has an internal diffraction grating and is coupled to a rotation stage controlled by a stepper motor and laptop computer (Fig. 5). In this pushbroom system, the laptop acquires 640 different skylight spectra at each rotation stage position (i.e., for each line of the resulting hyperspectral datacube). The Pika II’s 8 mm Schneider lens has a nominal field of view (FOV) \( \sim 33.4^{\circ} \), and each hyperspectral line of pixels subtends a linear angle \( \sim 0.045^{\circ} \) wide. The spectrometer measures spectral radiances \( L_\lambda \) from \( \lambda = 380–910 \) nm at a resolution of \( \sim 4.5 \) nm, and the camera’s analog-digital brightness resolution is 12 bits for each of its 120 spectral channels. In this paper, I analyze \( L_\lambda \) from 400–700 nm in 69 channels.

Spraybow and rainbow backgrounds are measured with the Pika II either directly (before the spray is turned on or when the rainbow disappears) or with a linear polarizer oriented radially to the bow at a particular \( \alpha \). I then measure solar-disc \( L_\lambda \) with a Photo Research PR-650 spectroradiometer that is fitted with a set of neutral density filters of known spectral transmissivity \( T_\lambda \). For this paper, I determine spray DSDs with an oil-immersion technique [31] described below. In later research, I plan to use a laser disdrometer that measures drop sizes and speeds in spray and rain, as well as giving the rainfall’s meteorological optical
range [32]. This range can be used to estimate a rainswath’s horizontal optical thickness.

Figure 5 illustrates my equipment setup for imaging primary and secondary spraybows with the Pika II spectrometer. The system’s hyperspectral camera is at image center, and it is attached to the rotation stage on the right, which precisely repositions the camera for each new scanline. This system can acquire a spectral datacube with pixel dimensions of 500 lines by 640 columns in a few seconds, and so it is well suited to long-lived spraybows. For the more ephemeral rainbows seen in showers, a much faster snapshot hyperspectral system is preferable [33].

Figure 6 shows hyperspectral results from my commercial spray head’s “fan” setting. Figure 6’s $L_\lambda$ spectra are of the colorimetrically purest red, green, and blue in Fig. 5’s primary, with each curve’s radiances averaged over $\Delta\alpha \sim 33^\circ$ or $N = 433–463$ pixels, depending on $r$. In Fig. 6, all $L_\lambda$ are for intrinsic spraybows (i.e., after the black background’s radiances are subtracted) produced at a constant water pressure, and the blue spectrum includes error bars two standard deviations wide centered on the mean $L_\lambda$. For Fig. 5’s setup, the background $L_\lambda$ are measured directly just before the spray is turned on, and care is taken not to wet the matte black background afterward.

Figure 6’s broad local maxima mean that even these purest spraybow colors are not especially pure, which provides hyperspectral confirmation of my earlier multispectral claim that natural rainbows are not color paragons [2]. Another illustration of this limited color gamut comes from Fig. 7, which separately plots spraybow $u’v’(r)$ for the spray head’s “fan” and “mist” DSDs, with the latter having distinctly fewer large drops (Fig. 8). Not surprisingly for the smaller-drop DSD, Fig. 7’s mist spray curve is perceptibly smaller than the fan spray curve. Yet my calculations show that even the fan spraybow’s gamut $\hat{g}$ [28] is only 5.9% of that for the spectrum locus (for which $\hat{g} = 1$) or $\sim 22%$ of that for the ColorCheckerDC, the card of color samples used to calibrate Figs. 1–4 [34]. Table 1 summarizes the observed chromaticity parameters for all of this paper’s primary rainbows, including their $\hat{g}$ and colorimetric areas $A$ spanned on the CIE diagram.

Unlike Fig. 2, in Fig. 7 the $u’v’(r)$ curves extend along the Planckian locus, a change that is independent of the UCS diagram’s slightly different (and anisotropic) scaling here. Thus Fig. 7 reconfirms what Figs. 5 and 6 show: in these high-sun spraybows, blues are much better defined visually and spectrally than are greens. In particular, note the narrower local maximum for Fig. 6’s blue spectrum compared with that for its green spectrum. As shown below, this behavior is not well explained by the Kön nen/Airy model. In Figs. 5 and 6, subtracting the spraybow’s black background is straightforward: its spatial distribution of $L_\lambda$ is measured immediately before starting the spray, and then these spectral radiances are subtracted pixel by pixel from the datacube of spray $L_\lambda$. Any hyperspectral pixels in this difference datacube found to have zero or negative radiances are excluded from the analysis.

For distant rain showers the process is more involved, although several techniques are useful. First, the rainbow may appear or disappear quickly enough that its distant cloud background changes minimally between hyperspectral datacubes which include or exclude the bow. In some of these cases, one can simply subtract pixel by pixel the latter datacube from the former. Second, the natural rainbow’s cloud background may be quite spatially uniform. In this case, experience indicates that within a limited $\alpha$ interval, one can use the average background $L_\alpha$ in Alexander’s dark band as a single, uniform background spectrum. This is the technique used in Figs. 2 and 4. Third, one can mount a linear polarizing filter on a hyperspectral camera and rotate the filter’s transmission axis to a particular rainbow clock angle $\alpha$, thereby greatly reducing rainbow $L_\lambda$ near that $\alpha$ [20,21,35]. Then one rotates the filter to $\alpha + 90^\circ$, which largely restores the rainbow image. Hyperspectral datacubes acquired at these two filter positions are used to spectrally calculate the rainbow signals minus their background radiances.

![Figure 6](image-url) Intrinsic $L_\lambda$ for spraybow colors having the purest red, green, or blue for Fig. 5’s fan spray DSD. Measurement intervals for $r$ and $\alpha$ are given in Table 1.

### Table 1. Chromaticity Parameters for Observed Primary Rainbows

<table>
<thead>
<tr>
<th>Figure</th>
<th>$b_0(\circ)$</th>
<th>$r$ Interval (°)</th>
<th>$\alpha$ Interval (°)</th>
<th>Droplet Source</th>
<th>Mean $u’$</th>
<th>Mean $v’$</th>
<th>Gamut $\hat{g}$</th>
<th>Area $A(\times 10^{-4})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 2</td>
<td>13.9</td>
<td>39.7–42.2</td>
<td>22.6–35.8</td>
<td>Thunderstorm</td>
<td>0.198562</td>
<td>0.481848</td>
<td>0.028999</td>
<td>3.0332</td>
</tr>
<tr>
<td>Fig. 4*</td>
<td>2.8</td>
<td>39.7–42.1</td>
<td>14.3–37.8</td>
<td>Thunderstorm</td>
<td>0.237728</td>
<td>0.509559</td>
<td>0.024695</td>
<td>1.8884</td>
</tr>
<tr>
<td>Figs. 6 and 7</td>
<td>61.9</td>
<td>38.7–44.0</td>
<td>78.8–111.7</td>
<td>Fan spray</td>
<td>0.210886</td>
<td>0.488396</td>
<td>0.058863</td>
<td>8.3530</td>
</tr>
<tr>
<td>Fig. 7</td>
<td>56.5</td>
<td>37.6–44.0</td>
<td>69.0–111.1</td>
<td>Fan spray</td>
<td>0.211087</td>
<td>0.489717</td>
<td>0.051568</td>
<td>6.2513</td>
</tr>
<tr>
<td>Figs. 11 and 12</td>
<td>9.8</td>
<td>41.1–43.8</td>
<td>1.0–21.6</td>
<td>Fan spray</td>
<td>0.223061</td>
<td>0.496562</td>
<td>0.035635</td>
<td>3.4432</td>
</tr>
</tbody>
</table>

*Maximum values of $\hat{g}$ and $A$ are in boldface, whereas minimum values are in italics.
4. MEASURING DROP SIZE DISTRIBUTIONS IN RAIN AND NEARBY SPRAY

Although a laser disdrometer can provide more detailed data on rain or spray DSDs, Fig. 8 gives nearby spray distributions with a simple manual technique based on [31]: a constant-pressure water spray briefly wets a Petri dish filled with ~2 cm of mineral oil that is thinned with a viscosity additive. I then promptly photograph each bottom-lit dish in darkness and digitally mask N > 500 identifiable drop outlines for each spray. Image-analysis software counts the two resulting relative DSDs. For spectral modeling purposes, such relative (rather than absolute) DSDs are acceptable because most rainbow scattering theories only calculate relative spectral intensities. Note that the near-uniform oil pressure surrounding a drop of any size makes it spherical. Thus one can only measure a large drop’s equivalent-volume radius rather than its original oblate spheroidal dimensions.

Stratiform and convective rain pose more complicated DSD measurement problems. Ideally, one should acquire many different DSDs that are measured by a spatially dense array of disdrometers that extends throughout the rainswath along each rainbow scattering angle. This measurement scheme is deemed ideal because small-scale rain DSDs vary in time, distance from the observer, and altitude [36–42]. Given that this ideal scheme is impractical, I consider some feasible alternatives. Used with care, these alternatives can provide realistic DSDs for simulating rainbow scattering over large optical paths in rain.

As Jameson and Kostinski note, “in statistically homogeneous rain, as the number of [rain] patches included increases, the observed spectrum of drop sizes approaches a ‘steady’ distribution” [39]. By statistically homogeneous rain, they mean rain with small-scale spatial variability in its DSDs but which nonetheless has a constant expected number of drops within the larger observation volume and thus an overall steady DSD, independent of the measurement type. This steady DSD is the kind that applies to a rainbow observer’s path-integrated scattering. Such DSDs can be approximated with exponential (e.g., Marshall–Palmer), gamma, or log-normal distributions given data on rain rates and types [43,44], and this is the approach used in Figs. 2 and 4 [27].

5. MEASURING SOLAR-DISC SPECTRA AS ILLUMINANTS FOR NATURAL RAINBOWS

Unlike rain DSDs, sunlight spectra can be measured and added to rainbow scattering models fairly readily. To do so, I attach a set of neutral density filters (median spectral transmissivity $T_{\lambda} \sim 3.0 \times 10^{-6}$) to the Photo Research PR-650 spectroradiometer. Given (a) independent measurements of the filter set’s $T_{\lambda}$ spectrum and (b) a measured correction factor that compensates for the sun slightly underfilling the PR-650’s 1°-diameter radiance FOV, I can measure spectrally corrected solar-disc $L_{\lambda}$ at the rainbow observer (see Fig. 9). Although this technique yields absolute (as opposed to relative) spectral solar-disc $L_{\lambda}$, both kinds of spectra are equally acceptable in rainbow simulations.

When modeling the spectra of nearby optically thin spraybows (typical spray optical thickness $\tau_{\lambda} < 0.02$), the local solar-disc $L_{\lambda}$ is unambiguously the spray drops’ illuminant. On the other hand, rainbow-producing rainswaths can be up to several kilometers away. Because incident sunlight rays are nearly parallel, they typically undergo very similar spectral extinction upon reaching both the observer and the distant rain. As a result, the rain’s incident solar-disc $L_{\lambda}$ likely differ little from those at the observer, but the extra optical pathlength from the rain to observer may perceptibly redden the measured rainbow’s $L_{\lambda}$ spectra. One way to address this minor rain-to-observer aerosol reddening is to subtract the observed rainbow’s background spectrum (which also includes the additional reddening) pixel by pixel to get the bow’s intrinsic spectrum, as is done in Figs. 2 and 4.
6. QUANTITATIVE COMPARISONS OF OBSERVED AND SIMULATED SPRAYBOWS

Figure 10 takes the environmental factors discussed in Sections 2–5 above and includes them in the Können/Airy model for oblate spheroidal drops. For Fig. 5’s optically thin, nearly horizontal spray, the spray drops’ oblate axes are approximately horizontal rather than vertical as they would be for raindrops. I call such drops H_oblate, and they travel in Fig. 5’s spray more slowly than their aerodynamic terminal speeds, which reduces their flattening by an amount proportional to the square of their speeds.

The two Können simulations in Fig. 10 take into account its fan spray drops’ (a) oblate axis orientations, (b) slower measured speeds (which reduce oblateness as a function of equivalent-volume drop size), and (c) measured DSD (see Fig. 8).

![Image](image_url)

Fig. 9. Solar-disc relative $L_\lambda$ measured at the earth’s surface as spraybow illuminants in Figs. 5–7, 11.

![Image](image_url)

Fig. 10. Primary fan-type spraybow observed for a high sun (see Figs. 5–7, 9) and simulated using the Können/Airy model [16].

Table 2. Best Fits Between Observed and Können/Airy Model Primary Rainbows (Fig. 10)

<table>
<thead>
<tr>
<th>Droplet Shape</th>
<th>$u'v'\Delta A$</th>
<th>$u'v'\Delta \hat{g}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_oblate</td>
<td>$1.047 \times 10^{-6}$</td>
<td>$8.00 \times 10^{-7}$</td>
</tr>
<tr>
<td>Vertical oblate</td>
<td>$1.032 \times 10^{-6}$</td>
<td>$1.12 \times 10^{-5}$</td>
</tr>
<tr>
<td>Spherical</td>
<td>$1.10 \times 10^{-6}$</td>
<td>$1.36 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Figure 10’s fit criterion is to calculate the smallest difference $\Delta A$ between the colorimetric areas $A$ spanned by the observed and modeled chromaticity curves on the same $r$ interval. This fit is achieved in Fig. 10 by scaling the measured background luminance $L_\nu$ before additively mixing the background and model $u'v'(r)$ at a given rainbow radius $r$. Such scaling is needed because the model’s $L_\nu(r)$ have arbitrary units, whereas the mean hyperspectral background $L_\nu$ has units of cd/m$^2$.

A different fit criterion scales background $L_\nu$ so that the two curves’ gamuts $\hat{g}$ match closely (i.e., it minimizes the gamut difference $\Delta \hat{g}$). Table 2 summarizes results for these two spraybow fit criteria and for three drop types (spherical drops are not shown in Fig. 10). Only $\Delta$ values within a given criterion are directly comparable, and smaller values indicate a better fit for the Können/Airy model.

Using either criterion, the resulting fit with Fig. 10’s fan spraybow’s observed hyperspectral $u'v'(r)$ chromaticity curve is not very good, especially for its enhanced blues. Changing from horizontal (square line markers) to vertical (dashed line) drop oblateness makes no significant change in the simulated chromaticity curve’s area, although in fact Fig. 5’s spray contains no vertical oblate drops. However, using H_oblate drops clearly does improve the $\hat{g}$ match in Table 2. The model $u'v'(r)$ curve for spherical drops is nearly indistinguishable from the curves for the two oblate types in Fig. 10, so I have omitted it to avoid graphical clutter.

Figure 11 juxtaposes observed $u'v'(r)$ curves for the same kind of fan spraybows at two different $b_0$. In Fig. 11, a radial chromaticity curve for $b_0 = 9.8\degree$ on 11 January 2016 is added to Fig. 10’s curve for $b_0 = 61.9\degree$ on 29 June 2015. Only the illuminant spectra differ for Fig. 11’s two curves: the spray DSD, measurement site, and black background (see Fig. 5) are all the same. My spraybows’ water source is a deep well fitted with a submersible pump, and its measured water temperatures differ by $<3\degree$C from June to January. Calculations with the Können model show that rainbow chromaticities and luminances produced by water drops at the extremes of this temperature range ($15\degree$C–$18\degree$C) are nearly identical. Furthermore, because spray droplets are exposed to the air only briefly, any seasonal changes in refractive index will be nearly nil.

As in Fig. 4, at the lower $b_0$ the entire $u'v'(r)$ curve shifts to lower color temperatures, with the MacAdam JND’s small size indicating a perceptible shift. Also typical of such low-sun bows, this primary’s gamut $\hat{g}$ is reduced from that for its high-sun counterpart, here by $\sim39\%$. Finally, the shape of the low-sun spraybow’s supernumerary hook near $r = 41.1\degree$ has changed, as has the rotational orientation of the entire chromaticity curve with respect to the Planckian locus. Yet despite these numerous quantitative colorimetric changes, my
and observed curves. Second, the simulated smaller than the same difference between the spherical-drop chromaticities.

Qualitative visual impression was that Fig. 11’s low-sun spraybow had changed negligibly.

Figure 12 repeats Fig. 10’s analysis for the low-sun spraybow. Using H_oblate rather than spherical drops in Fig. 12’s Können/Airy model significantly improves its fit to the observed \( u'v'(r) \) for primary \( r \text{ of } 41.1° - 43.8° \). (Axis aspect ratios differ in Figs. 10 and 12 in order to show as much relevant detail as possible.) First, the best-fit difference between the H_oblate and observed curves’ colorimetric areas \( A \) is ~76% smaller than the same difference between the spherical-drop and observed curves. Second, the simulated H_oblate curve’s shape and orientation much more closely resemble the observed spraybow (i.e., it is more congruent with the observed chromaticities).

Figure 13 shows how changes in \( h_0 \) affect the observed radial profiles of spraybow luminance \( L_v(r) \) in \( \text{cd/m}^2 \). Note that the low-sun \( L_v \) ordinate scale on the right axis is much smaller, as expected for its much weaker incident sunlight \( L_i \). Another important meteorological and optical difference between the high- and low-sun \( L_v(r) \) spraybow profiles is that their local maxima occur at different \( r \). The simplest qualitative explanation for this difference is that many fan-spray drops are not spheres, a fact consistent with the \( h_0 \)-dependent colorimetric changes seen in Figs. 11 and 12. Quantitatively explaining this shift of the maximum \( L_v(r) \) to larger \( r \) at lower \( h_0 \) is more complicated. However, my ray-tracing tests suggest that for H_oblate drops, the primary rainbow minimum-deviation angles actually decrease (and thus their supplements \( r_{\text{max}} \) increase) as \( h_0 \) decreases from 62° to 10°. Another factor that may contribute to the \( r_{\text{max}} \) shift in luminance is the reddening of sunlight at low \( h_0 \) (see Fig. 9).

Finally, note that Fig. 10’s \( u'v'(r) \) (chromaticity curves) are each calculated from 107 observed or simulated spraybow spectra. Each observed spectrum was measured at a different radius \( r \) in the 29 June 2015 fan spraybow, and collectively these 107 spectra span the radial interval \( r = 38.7° - 44.0° \) in 0.05° steps. Figure 14 plots two pairs of observed and H_oblate \( L_i \) spectra whose root-mean-square error (RMSE) differences have either the median or fifth-percentile values among these 107 spectral pairs. Each spectrum is normalized to have a sum \( = 1 \) so that differences in absolute or integrated \( L_i \) do not contribute to spectral differences in a given pair.

Because Fig. 14’s fifth-percentile spectral pair has the 400–700 nm RMSE value that is exceeded by 95% of all other pairs, it has one of the best spectral fits between the observed and Können high-sun spraybows in Fig. 10. In other words, the two \( u', v' \) coordinates at rainbow \( r = 38.8° \) are among Fig. 10’s closest chromaticities. Although spectral matching criteria other than RMSE are possible, the fact that it is directly related to measured and modeled colorimetric differences \( \Delta u'v'(r) \) makes it especially useful here.
Two spectral features are particularly noteworthy in Fig. 14. First, both spectral pairs exhibit large positive model-observed differences near 400 nm, which appears inconsistent with the Können/Airy theory spraybows at the specified percentiles. RMSE statistics are for comparisons of measured and modeled spectra at 107 spraybow radii from \( r = 38.7^\circ \)–44.0° in 0.05° steps.

7. CONCLUSIONS

Future research on natural rainbows should help me resolve some of the subtler problems that remain above, such as reconciling the colorimetric and spectral data on observed and simulated rainbows. Planned improvements include using the Sadeghi model’s asymmetrically flattened drops, since clearly droplet shapes can perceptibly change observed rainbow colors (e.g., Figs. 2, 4, and 10). Additional tools such as a laser disdrometer and hyperspectral snapshot camera will permit measurements of the rainfall environment and resulting spectra of natural rainbows that occur in short-lived rain showers.

But even at this early stage, the following points are clear: the colorimetric shapes and positions, visible features (e.g., primary and first supernumerary), and \( u'v'(r) \) trends of natural rainbows all have typical chromaticity patterns that are reasonably well described by a simple model of scattering by oblate spheroidal drops [16]. Although this Können/Airy model does not reproduce the pronounced blues in high-sun spraybows particularly well (Figs. 5 and 10), it does significantly better for low-sun rainbows and spraybows (Figs. 2, 4, and 12).

We now have spectral confirmation that, colorimetrically speaking, natural rainbows do not provide especially pure optical displays (Fig. 6). Finally, measured and modeled spectra of natural rainbows (Fig. 14) may not easily explain the resulting colorimetric differences.

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REFERENCES AND NOTES

33. At daytime lighting levels, snapshot hyperspectral cameras can acquire an angularly detailed spectral datacube in <1 sec, http://senop.fi/oportunics/hyperspectralCamera.
34. A successor color calibration card is available as the ColorChecker Digital SG, xritephoto.com/colorchecker-digital-sg.