I. WORK DONE BY FORCES

Consider the following cases

a)

\[ F_a \]

\[ m \]

\[ F_a \]

b)

c)

\[ F_a \]

\[ \theta \]

\[ m \]
\[ W = (\text{Force applied}) \cdot (\text{magnitude of displacement}) \]

\[ W = \vec{F} \cdot \Delta \vec{r} \]

Scalar product of two vectors

\[ \vec{A} \cdot \vec{B} = \]

\[ |\vec{A}| = \sqrt{9^2 + (4.5)^2} = 10.1 \]

\[ |\vec{B}| = 7 \]

\[ \vec{A} \cdot \vec{B} = (10.1)(7) \cos 26.6^\circ = 63 \]
II. Work Done by a Variable Force

If \( \vec{F}_x \) is not constant (i.e. spring force, etc.)

\[
W = \int_{x_0}^{x_f} F_x \, dx
\]

\[ W_c = F_x \Delta x_c \]

III. Work - Kinetic Energy

\[
W_{net} = \int (\sum \vec{F} \cdot d\vec{r})
\]

in one dimension

\[
W_{net} = \int \sum F \, dx
\]

Newton's 2nd Law
So,

\[ W_{net} = \int_{x_i}^{x_f} ma \, dx \]

But, \[ a = \frac{dv}{dt} \]

\[ W_{net} = \int_{x_i}^{x_f} m \frac{dv}{dt} \, dx = \int_{x_i}^{x_f} m \frac{dv}{dx} \frac{dx}{dt} \, dx \]

\[ = \int_{v_i}^{v_f} m \, v \, dv \]

\[ W_{net} = \int_{v_i}^{v_f} m \, v \, dv \]

Kinetic Energy \[ \Rightarrow \text{energy due to motion} \]

\[ K = \frac{1}{2}mv^2 \]

\[ W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta K \]

*Net work on system causes change in kinetic energy*
Example:

\[ F_a = 20N \]
\[ \theta = 120^\circ \]
\[ m = \text{-known} \]

- a) How fast is block moving after it has moved 10m?
- b) What is kinetic energy at that point?
- c) How much work was done on block?
IV. Work done by gravity

\[ y = y_{\text{top}} \quad k_{\text{top}} = \]

\[ y = 0 \quad v_0 \quad k_0 = \]

From ground to top

\[ W_{0 \rightarrow \text{top}} = \]

\[ \text{energy transferred out of system} \]

\[ F_g \downarrow \uparrow \Delta y \]

From top to ground

\[ W_{\text{top} \rightarrow 0} = \]

\[ \text{energy transferred in to system} \]

\[ F_g \downarrow \downarrow \Delta y \]