WORK AND KINETIC ENERGY II

I. WORK DONE BY GRAVITY

Push a block with mass 30 kg up a frictionless plane

\[ \theta = 15^\circ \]

a) What work did you do to the block to push it 3.0 m up the ramp at a constant speed?

\[ \Sigma F_x = F_a - mg \sin \theta = 0 \]

\[ F_a = mg \sin \theta \]

\[ W_a = \vec{F} \cdot \Delta \vec{r} = \int F_a \, d \cos \theta = (mg \sin \theta) \, d \]

\[ W_a = (30 \, kg)(9.8 \, m/s^2) \sin 15^\circ \times (3.0 \, m) \]

\[ W_a = 228 \, J \]
b) What work does gravity do over the same distance?

\[ W_g = \int \vec{F}_g \cdot d\vec{r} = (mg \sin \theta) \Delta \cos 180° \]

\[ = -228 J \]

\[ W_a + W_g = \Delta K = 0 \]

\[ W_g = -228 J \]

c) If you remove \( F_a \), what speed will the block have when it reaches its original position?

\[ W_g = \Delta K = 0 \]

\[ W_g = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = 0 \]

\[ v_f = \sqrt{\frac{2 W_g}{m}} = \sqrt{\frac{2(228)}{30}} \]

\[ \Rightarrow \quad v_f = 3.9 \text{ m/s} \]
II. WORK DONE BY A VARIABLE FORCE (SPRING FORCE)

\[ k = 10 \text{N/m} \]

\[ \begin{array}{c}
\text{F}_a \\
\text{m}
\end{array} \]

no friction

\[ 2\text{kg} \]

a) What work must you do to push the block and compress the spring \( d = 0.5\text{m} \)?

\[ \Sigma F_x = F_a - F_s = 0 \]

\[ F_a = F_s = \frac{1}{2} kx \]

\[ W_a = \int_{0}^{d} kx \text{F}_a \cdot dx = \int_{0}^{d} F \text{dx} \]

\[ = \int_{0}^{d} kx \text{dx} = \frac{1}{2} kx^2 \bigg|_{0}^{d} = \frac{1}{2} k(d^2 - 0) \]

\[ W_a = \frac{1}{2} k d^2 = \frac{1}{2} (10 \text{N/m})/(0.5\text{m})^2 \]

\[ W_a = 1.3 \text{J} \]
b) What work does the spring do?

\[ W_s = -W_q = -1.35 \]

c) If the compressed spring is let go, what speed does the block launch with?

\[ W_s = \Delta K \]

\[ W_s = \frac{1}{2}mv_f^2 \]

\[ v_f = \sqrt{\frac{2W_s}{m}} = \sqrt{\frac{2(1.35)}{2\text{kg}}} \]

\[ v_f = 1.1 \text{ m/s} \]
III. Power

\[ P_{avg} = \frac{\text{(Work Done)}}{\text{(Time it takes to do it)}} = \frac{W}{\Delta t} \]

Instantaneous power

\[ P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} \]

Power done by constant force

\[ P = \frac{dW}{dt} = \frac{d(F \cdot \Delta r)}{dt} = F \cdot \frac{d\dot{r}}{dt} = \vec{F} \cdot \vec{v} \]