Rotation of rigid bodies

This line is part of the body and perpendicular to the rotation axis.

The body has rotated counterclockwise by angle $\theta$. This is the positive direction.

This dot means that the rotation axis is out toward you.

$2\pi$ radians = 1 full revolution

Angular Position

$$\theta = \frac{s}{r} = \frac{\text{arc length}}{\text{radius}}$$

Angular Displacement

$\Delta \theta =$
Analogous to translational motion

Angular Position: \( \theta(t) \)

Angular Velocity: \( \omega(t) = \frac{d\theta(t)}{dt} \)

Angular Acceleration: \( \alpha(t) = \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2} \)

"Vector" rules for angular notation

\[ \text{ccw} \Rightarrow \Delta \theta > 0 \]

"Right hand rule"
Constant angular acceleration

If $\alpha(t) = \text{constant}$

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_0 + \alpha t$$

$$x_f = x_0 + v_0 t + \frac{1}{2} \alpha t^2$$

$$v_f = v_0 + \alpha t$$

Relating linear and angular variables

1. Angular position/displacement $\rightarrow$ Position

\[ s_1 = \]

\[ s_2 = \]
2. Angular speed \( \Rightarrow \) tangential speed

![Diagram showing the velocity vector is always tangent to the circle around the rotation axis.]

\[ \omega = \frac{d\theta}{dt} = \]

3. Angular acceleration \( \Rightarrow \) radial \& tangential acceleration

![Diagram showing the acceleration always has a radial (centripetal) component and may have a tangential component.]

\[ a_r = \frac{dv}{dt} = \]

\[ a_r = \]
Angular motion example

1. A submarine main turbine has a rotor radius of 2.0 m. If the turbines are initially spinning CCW at 900 rev/minute when a sharp metallic sound is heard, what angular acceleration is required to stop the blades in 1.5 s? How many revolutions do the blades go through before they stop if we assume the acceleration is constant?