**Simple Harmonic Motion**

Displacement: $X(t) = X_m \cos(\omega t + \phi)$

- $X_m$ = amplitude
- $(\omega t + \phi)$ = phase
- $\omega$ = angular frequency

Oscillations $\{ f, T \}$

Time

Cosine repeats every $2\pi$: $\omega T = 2\pi$

Frequency: $f = \frac{\omega}{2\pi}$
The amplitudes are different, but the frequency and period are the same. 

\[ x(t) = x_m \cos (\omega t + \phi) \]

This negative value shifts the cosine curve rightward. 

This zero gives a regular cosine curve.
\( x(t) = x_m \cos(\omega t + \phi) \)

\( v(t) = \frac{dx(t)}{dt} \)

\( a(t) = \frac{dv(t)}{dt} \)
Simple Harmonic Motion: Restoring Force

An example

\[ k \]

\[ -x_n \quad 0 \quad +x_n \]

\[ \text{no friction} \]

\[ \overrightarrow{F} = \overrightarrow{ma} \]

\[ \text{Diff. Eq.:} \]

\[ -kx = m \frac{d^2x}{dt^2} \]

\[ \frac{d^2x}{dt^2} = -\frac{k}{m}x \implies \text{what is only smooth function} \]

\[ \text{s.t.} \quad \frac{d^2f(t)}{dt^2} = f(t) \]

\[ \Rightarrow \text{Spring undergoes SHM!} \]

\[ -kx = ma = m(-\omega^2x) \]

so \[ k = m\omega^2 \]

\[ \omega = \sqrt{\frac{k}{m}} \]

\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{m}{k}}} \]

Period of a SHM (spring)
Energy in SHM (Spring Example)

\[ K(t) = \text{kinetic energy} = \frac{1}{2} m \left[ v(t) \right]^2 \]
\[ U(t) = \text{stored spring energy} = \frac{1}{2} k \left[ x(t) \right]^2 \]

So:

\[ K(t) = \frac{1}{2} m \omega^2 x_m^2 \sin^2 (\omega t + \phi) = \frac{1}{2} k x_m^2 \sin^2 (\omega t + \phi) \]
\[ U(t) = \frac{1}{2} k x_m^2 \cos^2 (\omega t + \phi) \]

**Total Energy:**

\[ E = K + U \]
\[ = \frac{1}{2} k x_m^2 \left[ \sin^2 (\omega t + \phi) + \cos^2 (\omega t + \phi) \right] \]
\[ E = \frac{1}{2} k x_m^2 \]

As time changes, the energy shifts between the two types, but the total is constant.

As position changes, the energy shifts between the two types, but the total is constant.