23. Write out the total wave function $\Psi(x, t)$ for an electron in the $n = 3$ state of a 10 nm wide infinite well. Other than the symbols $x$ and $t$, the function should include only numerical values.

For an infinite square well, the spatial solution is

$$\Psi_n(x) = \sqrt{\frac{2}{L}} \sin k_n x \quad \text{inside}$$

$$\Psi(x) = 0 \quad \text{outside}$$

Since $n = 3$ and $L = 10$ nm, we find $k_n = \frac{2\pi}{L} = \frac{3\pi}{10 \times 10^{-9} \text{m}}$

$$k_3 = 9.42 \times 10^8 \text{ rad/m}$$

so the spatial part is $\Psi_3(x) = \sqrt{\frac{2}{10 \times 10^{-9} \text{m}}} \sin (9.42 \times 10^8 x)$

Now the time part has $E$ in $\alpha t$, so we must find $E$. For a particle in a box $E_n = \frac{1}{2m} \left( \frac{n\pi \hbar}{L} \right)^2$.

Here, $n = 3$

$$E_3 = \frac{1}{2m} \left( \frac{3\pi}{9.11 \times 10^{-31} \text{kg}} \right)^2 \left( 3 \frac{\pi}{10 \times 10^{-9} \text{m}} \right)^2 \approx 5.42 \times 10^{-21} \text{J}$$

So $\frac{E_3}{\hbar} = 5.42 \times 10^{21} \text{ J/s}$

All together

$$\Psi(x, t) = \Phi(t) \Psi(x) = 1.41 \times 10^4 \text{ m}^{-1/2} e^{-i \left( 5.14 \times 10^{13} \text{ rad/s} \right) t} \sin \left( 9.42 \times 10^8 \text{ rad/m} x \right)$$

I used 3 sig figs, but the problem is ambiguous. I will accept 1-3 sig figs. (b/c of the answer in text.)
An electron in the \( n = 4 \) state of a 5 nm wide infinite well makes a transition to the ground state, giving off energy in the form of a photon. What is the photon's wavelength?

The energy levels for a particle in an infinite well is given by \( E_n = \frac{1}{2m} \left( \frac{n+\frac{1}{2}}{L} \right)^2 \).

The conservation of energy tells us the energy lost by the electron-well system goes into the photon, so the photon's energy is

\[
E_{\text{photon}} = E_n - E_1 = \frac{\pi^2 \hbar^2}{2mL^2} (4^2 - 1^2) = \frac{15\pi^2 \hbar^2}{2mL^2}
\]

and according to Einstein a photon's energy is given by \( E_{\text{photon}} = hf = \frac{hc}{\lambda} \).

So \( \lambda = \frac{hc}{E_{\text{photon}}} = \frac{hc}{\left( \frac{2mL^2}{15\pi^2 \hbar^2} \right)} = \frac{8\pi^2 mL^2 c}{15\pi^2 \hbar} \).

\[
\lambda = \frac{8 mL^2 c}{15 \hbar} = \frac{8 \left( 9.11 \times 10^{-31} \text{kg} \right) \left( 5 \times 10^{-9} \text{m} \right)^2 \left( 3.00 \times 10^8 \text{m/s} \right)}{15 \times 6.62 \times 10^{-34} \text{ J} \cdot \text{s}}
\]

\[
\lambda = 5.5 \times 10^{-6} \text{ m} = 5500 \text{ nm (infrared)}
\]

should be 1 sig fig, but I will accept 1 or 2.