14. Verify that the transmission and reflection probabilities given in equation (6-7) add to 1.

\[ T = \frac{4 \sqrt{E(E-U_0)}}{(\sqrt{E} + \sqrt{E-U_0})^2} \quad \text{and} \quad R = \frac{(\sqrt{E} - \sqrt{E-U_0})^2}{(\sqrt{E} + \sqrt{E-U_0})^2} \]

\[ T + R = \frac{4 \sqrt{E(U_0-E)} + (\sqrt{E} - \sqrt{E-U_0})^2}{(\sqrt{E} + \sqrt{E-U_0})^2} \]

\[ T + R = \frac{4 \sqrt{E(U_0-E)} + E - 2\sqrt{E(U_0-E)} + E-U_0}{(\sqrt{E} + \sqrt{E-U_0})^2} \]

\[ T + R = \frac{2E + 2\sqrt{E(U_0-E)} - U_0}{E + 2\sqrt{E(U_0-E)} + E-U_0} = \frac{2E + 2\sqrt{E(U_0-E)} - U_0}{2E + 2\sqrt{E(U_0-E)} - U_0} \]

\[ T + R = 1 \quad \checkmark \]

15. Calculate the reflection probability for a 5 eV electron encountering a step in which the potential drops by 2 eV.

\[ E = 5 \text{ eV} \]

\[ U = 0 \quad U_0 = -2 \text{ eV} \]

Step down means \( U_0 = -2 \text{ eV} \) is negative. Otherwise, we use normal reflection probability for a step potential.

\[ R = \frac{(\sqrt{E} - \sqrt{E-U_0})}{(\sqrt{E} + \sqrt{E-U_0})^2} \]

\[ R = \frac{\left( \frac{\sqrt{5}-\sqrt{7}}{\sqrt{5}+\sqrt{7}} \right)^2}{\left( \frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} \right)^2} = \left( \frac{-0.4097}{4.1882} \right)^2 = 7.04 \times 10^{-3} \]

\[ R = 0.7 \% \] to one sig. fig.

Bob gives \( R = 0.00704 \). I will take it to 3 sig. figs.
17. A beam of particles of energy $E$ and incident upon a potential step of $U_0 = \frac{2}{4} E$ is described by the wave function

$$\psi_{\text{inc}}(x) = 1 e^{i k x}$$

(a) Determine completely the reflected wave and the wave inside the step by enforcing the required continuity conditions to obtain their (possibly complex) amplitudes.

$$\psi_{x<0} = A e^{i k x} + B e^{-i k x} \quad (\text{eq} \text{ 6.4})$$

$$\psi_{x>0} = C e^{-\alpha x} \quad (\text{Eq. } 6.10)$$

so $A = 1$

The boundary conditions (see page 201)

$$A + B = C$$

and

$$i k (A - B) = -\alpha C$$

$$1 + B = C$$

and

$$i k (1 - B) = -\alpha C$$

Eliminate $C$:

$$ik (1 - B) = -\alpha (1 + B)$$

$$ik - ik B = -\alpha - \alpha B$$

$$(k - ik) B = -\alpha - ik$$

$$B = -\frac{(k + ik)}{(k - ik)}$$

$$B = -\frac{\sqrt{2mE \hbar}}{\hbar} \left( \frac{1}{2} + i \right)$$

$$B = \frac{3}{5} - \frac{4}{5}i$$

so

$$\psi_{\text{ref}} = (\frac{3}{5} - \frac{4}{5}i) e^{-i k x}$$

$$C = B + 1 = \frac{8}{5} - \frac{4}{5}i$$

so

$$\psi_{\text{trans}} = (\frac{8}{5} - \frac{4}{5}i) e^{-\alpha x}$$

(b) Verify by explicit calculation that the ratio of reflected probability density to the incident probability density is 1.

$$R = \frac{B^* B}{A^* A} = \frac{B^* B}{1} = \left( \frac{3}{5} + \frac{4}{5}i \right) \left( \frac{3}{5} - \frac{4}{5}i \right) = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$$

$$|R| = 1$$ as expected \(\checkmark\)