21. What fraction of a beam of 50 eV electrons would get through a 200 V, 1 nm wide electrostatic barrier?

\[
E = 50 \text{ eV} \\
U_0 = qV = 200 \text{ eV} \\
E < U_0
\]

\[
L = \sqrt{\frac{2m(U_0 - E)}{k}} = \sqrt{\frac{2(9.11 \times 10^{-31} \text{kg}) (200 - 50) \times 1.6 \times 10^{-19} \text{J/eV}}{1.095 \times 10^{-34} \text{J/s}}} \\
= 6.27 \text{ nm, seems wide.}
\]

\[
T = 16\left(\frac{50}{200}\right) (1 - \frac{50}{200}) e^{-2(6.27)} = \sqrt{1.1 \times 10^{-54}}
\]

Extra sig fig is OK.

24. Consider a potential barrier of height 30 eV. (a) Find a width around 1.000 nm for which there will be no reflection of 35 eV electrons incident upon the barrier.

(b) What would be the reflection probability for 36 eV electrons incident upon the same barrier? (Note: This corresponds to a difference in speed of less than 1%.)

Solve for \( \frac{N}{L} \):

\[
\left(\frac{N}{L}\right)^2 = \frac{2m}{k} \left( E - U_0 \right)
\]

\[
N = \sqrt{\frac{2m(E - U_0)}{k}}
\]

\[
= 3.64 \times 10^9
\]

If \( L \approx 1 \text{ nm} \), then \( N = 3.64 \) of course \( n \) is integer so round up to: \( n = 4 \), then \( L = 1.09 \text{ nm} \)

(a) Reflection probability (Eq. 6-13)

\[
R = \frac{\sin^2 \left( \sqrt{\frac{2m(E_0 - U_0)}{k}} \frac{L}{n} \right)}{\sin^2 \left( \sqrt{\frac{2m(E_0 - U_0)}{k}} \frac{L}{n} \right) + 4 \left( \frac{E_0 - U_0}{U_0} \right) \left( \frac{E_0 - U_0}{U_0 - n} \right)}
\]

\[
= \frac{2.911 \times 10^{-31} \text{kg} (36 - 30) \times 1.6 \times 10^{-19} \text{J/eV}}{1.095 \times 10^{-34} \text{J/s}}
\]

\[
= 13.76
\]

Remember to use radians in radians

\[
\frac{\sin^2(13.76)}{\sin^2(13.76) + 4 \left( \frac{36}{30} \right) \left[ \frac{36}{30} - 1 \right]} = 0.48
\]

No even close to zero!
32. Jump to Jupiter  The gravitational potential energy of a 1 kg object is plotted versus position from Earth's surface to the surface of Jupiter. Mostly it is due to the Sun, but there are downturns at each end, due to the attractions to the two planets.

\[ L = 6 \times 10^{11} \text{ m} \]
\[ V_0 = 4 \times 10^8 \text{ J/kg} \]
\[ U_0 = mV_0 = 65 \text{ kg} \cdot 4 \times 10^8 \text{ J/kg} \]
\[ U_0 = 2.6 \times 10^{10} \text{ J} \]
\[ K = \frac{1}{2} m v^2 = \frac{1}{2} (65 \text{ kg}) (4 \text{ m/s})^2 = 520 \text{ J} \]
\[ E = K + U_0 = K \text{ at surface of earth} \]
\[ E < U_0 \]

This is a wide barrier, but let's check:

\[ L = \sqrt{2m(U_0 - E)} \]
\[ = \sqrt{2 \cdot 65 \text{ kg} \cdot (2.6 \times 10^{10} \text{ J} - 5.2 \times 10^2 \text{ J})} \]
\[ = 1.046 \times 10^5 \text{ m} \]

\[ \frac{L}{8} = 1.046 \times 10^5 \text{ m} \]

Very wide

\[ T = 16 \left( \frac{E}{U_0} \right) \left( 1 - \frac{E}{U_0} \right) e^{-2L/8} \text{ (Eq 6.18, wide barrier)} \]
\[ T = 16 \left( \frac{5.2 \times 10^2 \text{ J}}{2.6 \times 10^{10} \text{ J}} \right) \left( 1 - \frac{5.2 \times 10^2 \text{ J}}{2.6 \times 10^{10} \text{ J}} \right) e^{-2 \left( 1.046 \times 10^5 \text{ m} \right)} \]
\[ T \approx 3.2 \times 10^{-7} \text{ } e^{-2 \times 10^5} \approx 0 \text{ very very tiny!} \]
Fusion in the Sun: Without tunneling, our Sun would fail us. The source of its energy is nuclear fusion, and a crucial step is the fusion of a light-hydrogen nucleus, which is just a proton, and a heavy-hydrogen nucleus, which is of the same charge but twice the mass. When these nuclei get close enough, their short-range attraction via the strong force overcomes their Coulomb repulsion. This allows them to stick together, resulting in a reduced total mass/internal energy and a consequent release of kinetic energy. However, the Sun’s temperature is simply too low to ensure that nuclei move fast enough to overcome their repulsion.

(a) By equating the average thermal kinetic energy that the nuclei would have when distant, \( \frac{3}{2} k_b T \), and the Coulomb potential energy they would have when 2 fm apart, roughly the separation at which they stick, show that a temperature of about \( 10^9 \) K would be needed.

\[
\frac{3}{2} k_b T = \frac{k e^2}{r} = \frac{2}{3} \left( \frac{8.99 \times 10^9 \text{ N m}^2 \text{C}^2}{(1.6 \times 10^{-19} \text{ C})^2} \right) \left( 1.38 \times 10^{-23} \text{ J/K} \right) (2 \times 10^{-15} \text{ m})
\]

\[
T = 6 \times 10^9 \text{ K}
\]

(b) In wide barrier the factor in exponential is

\[
\frac{2L}{b} = a \sqrt{2m(U_0 - E)} \frac{A + 1}{h}
\]

where \( E = \frac{k e^2}{b} \)

so

\[
U_0 - E = \frac{2k e^2}{b} - \frac{k e^2}{b} = \frac{k e^2}{b} = 4 \left( \frac{3}{2} k_b T \right)
\]

and

\[
b = 2k e^2 = L
\]

(c) Using the proton mass for \( m \), evaluate this factor a temperature of \( 10^7 \) K. Then evaluate it at 3000 K about that of an incandescent filament or hot filament and rather high by Earth standards. Discuss the consequences.
\[ \frac{2L}{\delta} = \frac{2\sqrt{2m \left[ 4 \left( \frac{3}{2}k_BT \right) \right]}}{h} \frac{2ke^2}{4kBT} \]

\[ \frac{2L}{\delta} = \frac{k_0e^2}{h} \sqrt{\frac{4m}{3k_BT}} \quad \text{and} \quad k = \frac{1}{4\pi\epsilon_0} \]

So \[ T \propto \exp \left( -\frac{2L}{\delta} \right) = \exp \left( -\frac{e^2}{4\pi\epsilon_0 h} \sqrt{\frac{4m}{3k_BT}} \right) \checkmark \]

(C) \[ a + T = 1 \times 10^7 \text{ } k \]

\[ \exp \left( -\frac{2L}{\delta} \right) = \exp \left( -\frac{(1.6 \times 10^{-19})^2 (8.99 \times 10^9 \text{ Nm}^2/\text{C}^2)}{1.055 \times 10^{-34} \text{ J s}} \frac{4 \times 1.67 \times 10^{-27} \text{ kg}}{3 \times 1.38 \times 10^{-23} \text{ J} / \text{K}^\circ} \right) \]

\[ T \approx 1.6 \times 10^{-4} \text{ small, but still manageable} \]

\[ a + T = 3000 \text{ } k \]

\[ \exp \left( -\frac{2L}{\delta} \right) = \exp \left( -\sqrt{\frac{2 \times 10^{-220}}{2 \times 10^{-220}}} \right) = 2 \times 10^{-220} \text{ never going to happen!} \]