11. For small \( z \), \( e^z \) is approximately \( 1 + z \). (a) Use this to show that Planck's spectral energy density (3-1) agrees with the result of classical wave theory in the limit of small frequencies. (b) Show that, whereas the classical formula diverges at high frequencies—the so-called ultraviolet catastrophe of this theory—Planck's formula approaches 0.

(a) Start with Eq 3-1 (p. 75):

\[
\frac{dU}{df} = \frac{hf}{e^{hf/kT} - 1} \cdot \frac{8\pi NV}{c^3} f^2
\]

Now use approximation for exponential function:

\[ e^{hf/kT} \approx 1 + \frac{hf}{kT} \quad \text{(good if } hf \ll kT \text{ or small frequencies)} \]

So denominator becomes:

\[ e^{hf/kT} - 1 \approx 1 + \frac{hf}{kT} - 1 = \frac{hf}{kT} \]

And:

\[
\frac{dU}{df} \approx \frac{hf}{hf/kT} \cdot \frac{8\pi NV}{c^3} f^2 = \frac{kT \cdot 8\pi NV}{c^3} f^2 \quad \checkmark
\]

This exactly the classical expression on page 74

(b) Now consider \( \lim_{f \to \infty} \) in Planck's function:

\[
\lim_{f \to \infty} \frac{dU}{df} \propto \frac{f^3}{e^{hf/kT} - 1} \to \infty \quad \text{both numerator and denominator approach infinity}
\]

However \( f^3 \) approaches \( \infty \) slower than \( e^{hf/kT} \) so the whole fraction goes to zero.

If you prefer to be formal, apply L'Hôpital's rule 3 times:

\[
\lim_{f \to \infty} \frac{dU}{df} \propto \frac{\text{constant}}{e^{hf/kT}} \to 0 \quad \checkmark
\]
12. At what wavelength does the human body emit the maximum electromagnetic radiation? Use Wien's law from Exercise 14 and assume a skin temperature of 70°F.

From exercise 14, we have \( \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{T} \)

\( T = 70^\circ \text{F} \) we must convert to Kelvin, first Celsius:

\( T_{\text{celsius}} = \frac{(\text{Fahrenheit} - 32)}{\text{9/5}} = \frac{(70 - 32)}{\text{9/5}} = 21^\circ \text{C} \)

Then \( T_{\text{Kelvin}} = (T_{\text{celsius}} + 273) \text{K} = 21 + 273 = 294 \text{ K} \)

\( \lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{294 \text{ K}} = 9.686 \times 10^{-6} \text{ m} = 9860 \text{ nm} \)

Remember Roy G. Biv

\( 700 \text{ nm} \uparrow \)

\( 400 \text{ nm} \downarrow \)

So 9860 nm is in the infrared.

This should be one sig. fig. I left three to make it easier for me to grade. I will accept up to three, but I'm not happy about it.
Section 3.2

16. In the photoelectric effect, photoelectrons begin leaving the surface at essentially the instant that light is introduced. If light behaved as a diffuse wave and an electron at the surface of a material could be assumed localized to roughly the area of an atom, it would take far longer. Estimate the time lag, assuming a work function of 4 eV, an atomic radius of approximately 0.1 nm, and a reasonable light intensity of 0.01 W/m².

The "area" of the electron is \( A = \pi r^2 = \pi (0.1 \times 10^{-9} \text{m})^2 \)

\[ A = 3.14 \times 10^{-20} \text{ m}^2 \]

The power delivered is intensity times area

\[ P_{\text{delivered}} = IA = 0.01 \text{ W/m}^2 \times 3.14 \times 10^{-20} \text{ m}^2 = 3.14 \times 10^{-22} \text{ J/s} \]

The energy required to free the electron is \( \phi = 4 \text{ eV} \)

And \( \phi = 4 \times 1.602 \times 10^{-19} \text{ J} = 6.41 \times 10^{-19} \text{ J} \)

So the time to build up this much energy is

\[ t = \frac{\phi}{P_{\text{delivered}}} = \frac{6.41 \times 10^{-19} \text{ J}}{3.14 \times 10^{-22} \text{ J/s}} = 2.0 \times 10^{-3} \text{ s} \]

This is about a half hour lag. No such lag is observed, we conclude the wave model does not work for the photoelectric effect.

20. What is the wavelength of a 2.0 mW laser from which \( 6 \times 10^{15} \) photons emanate every second?

The power is the energy of a photon times the number of photons emitted per unit time

\[ P = N \frac{h\nu}{\lambda} = \frac{NhC}{\lambda} \quad \text{where} \quad E = h\nu = \frac{hC}{\lambda} \quad \text{is the energy of a photon} \]

\[ \lambda = \frac{NhC}{P} = \frac{(6 \times 10^{15} \text{s}^{-1}) (6.63 \times 10^{-34} \text{J} \cdot \text{s}) (3 \times 10^8 \text{m/s})}{2.0 \times 10^{-3} \text{ J/s}} \]

\[ \lambda = 5.97 \times 10^{-7} \text{ m} = 597 \text{ nm} \]

This is in the yellow part of the spectrum.
24. With light of wavelength 520 nm, photoelectrons are ejected from a metal surface with a maximum speed of $1.78 \times 10^8$ m/s. (a) What wavelength would be needed to give a maximum speed of $4.81 \times 10^8$ m/s? (b) Can you guess what metal it is?

Max $KE$ is the photon's energy minus the work function (energy to free loosely bound electron).

$$K_{max} = hf - \phi$$

Solve for $\phi$

$$\phi = hf - K_{max}$$

$$f = c/\lambda \quad \text{so}$$

$$\phi = \frac{hc}{\lambda} - K_{max} \quad \text{as long as } U < c \text{ then } k = \frac{1}{2}mv^2$$

$$\phi = \frac{hc}{\lambda} - \frac{1}{2}meV^2 = \frac{(6.63 \times 10^{-34} J \cdot s)(3.00 \times 10^8 m/s)}{520 \times 10^{-9} m} - \frac{1}{2} \left(9.11 \times 10^{-31} kg\right) \left(4.81 \times 10^8 m/s\right)^2$$

$$\phi = 3.68 \times 10^{-19} J \quad \text{This depends on the metal so it is constant.}$$

So now solve for $\lambda$!

$$\frac{hc}{\lambda} = \phi + \frac{1}{2}meV^2$$

$$\lambda = \frac{hc}{\phi + \frac{1}{2}meV^2} = \frac{(6.63 \times 10^{-34} J \cdot s)(3.00 \times 10^8 m/s)}{3.68 \times 10^{-19} J + \frac{1}{2}(9.11 \times 10^{-31} kg)(4.81 \times 10^8 m/s)^2}$$

$$\lambda = 4.20 \times 10^{-7} m = 420 \text{ nm}$$

(b) See Table 3.1 on p. 76, the wave functions are in ev so convert

$$\phi = 3.68 \times 10^{-19} J \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} J} = 2.3 \text{ eV}$$

This matches Sodium.