31. A 0.057 nm X-ray photon "bounces off" an initially stationary electron and scatters with a wavelength of 0.061 nm. Find the directions of scatter of (a) the photon and (b) the electron.

\[ \frac{\lambda}{\lambda'} = \frac{h}{\lambda'} \cos \theta + \gamma mu \cos \phi \]
\[ 0 = \frac{h}{\lambda'} \sin \theta - \gamma mu \sin \phi \]

Compton effect
\[ \lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta) \]

Use Compton effect to find \( \theta \) (for photon)
\[ \cos \theta = 1 - \frac{mc(\lambda' - \lambda)}{h} = 1 - \frac{9.11 \times 10^{-31} (3 \times 10^8 \text{m/s})(0.061 - 0.057) \times 10^{-9} \text{m}}{0.53 \times 10^{-39} \text{J/s}} \]
\[ \cos \theta = -0.6488 \]
\[ \theta = 130.45^\circ \approx \boxed{130.5^\circ} \] (should really be \( \pm \) sig figs)

(b) Now use consv. of momentum to find \( \phi \) (for electron)
\[ \gamma mu \sin \phi = \frac{h}{\lambda'} \sin \theta \]
\[ \gamma mu \cos \phi = h \left( \frac{1}{\lambda'} - \frac{1}{\lambda} \cos \theta \right) = h \left( \frac{\lambda' - \lambda \cos \theta}{\lambda'} \right) = \frac{h}{\lambda'} \left( \frac{\lambda' - \lambda \cos \theta}{\lambda'} \right) \]

Divide:
\[ \tan \phi = \frac{\lambda \sin \theta}{\lambda' - \lambda \cos \theta} = \frac{0.057 \sin 130.5^\circ}{0.061 - 0.057 \cos 130.5^\circ} = 0.4448 \]
\[ \phi = \tan^{-1}(0.4448) = \boxed{23.9^\circ} \] (should really be \( \pm \) sig figs)
41. A stationary muon $\mu^-$ annihilates with a stationary antimuon $\mu^+$ (same mass, $1.88 \times 10^{-28}$ kg, but opposite charge). The two disappear, replaced by electromagnetic radiation. (a) Why is it not possible for a single photon to result? (b) Suppose two photons result. Describe their possible directions of motion and wavelengths.

(a) Before annihilation, neither is moving so $P_{\text{tot}} = 0$. A single photon cannot have zero momentum, so there must be at least 2 photons.

(b) They would move in opposite directions. They would have to have the same magnitude of momentum to cancel.

From mass:

$P < P_-$ must cancel

Conservation of $E$ requires

$2mc^2 = 2 \frac{hc}{\lambda}$

$a = \frac{h}{mc} = \frac{6.63 \times 10^{-34} \text{ J s}}{1.88 \times 10^{-28} \text{ kg} \cdot 3 \times 10^8 \text{ m/s}} = 1.18 \times 10^{-14} \text{ m}$

44. A beam of 500 nm light strikes a barrier in which there is a narrow single slit. At the very center of a screen beyond the single slit, $10^{12}$ photons are detected per square millimeter per second. (a) What is the intensity of the light at the center of the screen? (b) A second slit is now added very close to the first. How many photons will be detected per square millimeter per second at the center of the screen now?

(a) Find energy of a single photon $E = \frac{hc}{\lambda} = \frac{3.978 \times 10^{-19} \text{ J}}{500 \text{ nm}}$.

Intensity is energy per unit area per unit time. Convert:

$\frac{10^{12} \text{ photons}}{\text{mm}^2 \cdot \text{s}} \cdot \frac{(1000 \text{ mm})^2}{1 \text{ m}^2} = 10^{18} \frac{\text{photons}}{\text{m}^2 \cdot \text{s}}$

So $I = (3.978 \times 10^{-19} \text{ J}) \times 10^{18} = 0.398 \text{ W/m}^2$

(b) If you are interested, at the center, the waves are in phase and add so that $E = 2E_0 \sin(kx - wt)$. Intensity is $I \propto E^2$, so $I = 4I_0$. Intensity is 4 times higher, but energy of single photon is unchanged. Thus, means rate must increase by $4$.

$4 \times 10^{12} \frac{\text{photons}}{\text{mm}^2 \cdot \text{s}}$