2.24 A pole-vaulter holds a 16 ft. pole. A barn has doors at both ends, 10 ft. apart. The pole-vaulter on the outside of the barn begins running toward one of the open barn doors, holding the pole level in the direction he's running. When passing through the barn, the pole fits (barely) entirely within the barn all at once. (a) How fast is the pole-vaulter running? (b) According to whom — the pole-vaulter or an observer stationary in the barn — does the pole fit in all at once? (c) According to the other person, which occurs first, the front end of the pole leaving the barn or the back end entering, and (d) what is the time interval between these two events?

(a) This a length contraction problem.

10 Remember: proper length is the longest.

So \( L = \frac{L_0}{\gamma} \) proper length

\( \gamma > 1 \) greater than (or equal to) 1

\( L = 10 \text{ ft} \)

\( L_0 = 16 \text{ ft} \)

\( \gamma = \frac{L_0}{L} = \frac{16}{10} = 1.6 \)

\( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \) (p. 116)

\( \gamma^2 = \frac{1}{1 - \frac{v^2}{c^2}} \)

\( 1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2} \)

\( 1 - \frac{1}{\gamma^2} = \frac{v^2}{c^2} \)

\( c^2 (1 - \gamma^2) = v^2 \)

\( v = c \sqrt{1 - \frac{1}{1.6^2}} = c \sqrt{1 - \frac{1}{1.6^2}} = 0.78c \)

\( v = 0.78c = 2.3 \cdot 10^8 \text{ m/s} \)

(b) The observer in the barn sees the pole short enough for doors to close simultaneously.
Front end enters and leaves before back end enters.

(d) This is not time dilation because the two events—leaving of front end and entering of back end—do not occur at the same location. However, the two events are simultaneous in the barn's frame, so \( \Delta t = 0 \).

In the moving frame, we use eq. 2.126 twice:

\[ t' = \gamma \left( -\frac{v}{c^2} x + t \right) \]

\[ t'_{f} - t'_{i} = \gamma \left[ \left( -\frac{v}{c^2} x_{f} + t_{f} \right) - \left( -\frac{v}{c^2} x_{i} + t_{i} \right) \right] \]

\[ t'_{f} - t'_{i} = \gamma \left( \frac{v}{c^2} x_{f} + t_{f} - t_{i} \right) \]

\[ t'_{f} - t'_{i} = \gamma \left( \frac{v}{c^2} l \right) \]

\[ = 1.6 \left( \frac{0.78}{c} \right) 3.048 \text{ m} = 11.27 \times 10^{-8} \text{ s} \]

So the back leaves 11.27 ns after the front leaves according to runner in the primed frame.
Anna and Bob are in identical spaceships, each 100 m long. The diagram shows Bob’s view as Anna’s ship passes him at 0.8c. Just as the backs of the ships pass one another, both clocks there read 0. At the instant shown, Bob Jr., on board Bob’s ship, is aligned with the very front of Anna’s ship. He peers through a window in Anna’s ship and looks at the clock there.

(a) In relation to his own ship, where is Bob Jr.? (b) What does the clock he sees on Anna’s ship read?

(a) This is a length contraction problem. Bob sees Anna’s ship contracted.

\[ L = \frac{L_0}{\gamma} \]

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - (0.8)^2}} = 1.67 \]

\[ L = \frac{100 \text{ m}}{1.67} = 60 \text{ m} \]

(b) Bob and Bob Jr.’s clock must agree. So Bob Jr. cannot read 0 on Anna’s clock. The time he reads comes from Eq. 2.12b:

\[ t' = \gamma (\frac{\gamma}{c^2} x + t) \]

Where \( x = 60 \text{ m} \) and \( t = 0 \)

\[ t' = 1.67 (-0.8 \times 60 \text{ m}) = -2.67 \times 10^{-7} \text{ s} \]

\[ \approx -2.7 \text{ ns} \]

2.33 A famous experiment detected 527 muons per hour at the top of Mt. Washington, New Hampshire, elevation 1910 m. At sea level, the same equipment detected 395 muons per hour. The detector was sensitive to muons with speeds between 0.9950c and 0.9954c. Given that the mean lifetime \( \tau \) of a muon in its rest frame is 2.2 \( \mu \)s and that in this frame the number of muons decays exponentially with time according to \( N = N_0 e^{-t/\tau} \), show that the results obtained in this experiment are reasonable.

We can approach this using time dilation or length contraction. (see Ex. 2.2 p. 18)

I will use time dilation. In the experimenters frame: \( \Delta t = \gamma \Delta t_0 \)

Or \( \tau = \gamma \tau' \) is mean lifetime (longer than \( \tau' \))

\( \nu_{avg} = 0.9952c \) so \( \gamma_{avg} = \frac{1}{\sqrt{1 - (0.9952)^2}} = 10.218 \)

\( \tau = \gamma \tau' = (10.218) (2.2 \times 10^{-6} \text{ s}) = 2.2 \times 10^{-5} \text{ s} \) in experimenters frame

In experimenters frame \( \Delta t = \frac{\Delta x}{\nu} = \frac{910 \text{ m}}{0.9952c} = 6.397 \times 10^{-5} \text{ s} \)

\[ \frac{N}{N_0} = e^{-6.397 \times 10^{-6} / 2.2 \times 10^{-5}} = 0.75 \]

Theoretically, and \( \frac{N}{N_0} = \frac{395}{527} = 0.75 \)

Ratio found experimentally matches that found theoretically.