2. The transverse wave equation for a wave traveling on a string is \( \frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2} \).

Show by direct substitution that \( y = \left( t \frac{x}{c} \right)^2 \) is or is not a solution to the wave equation.

\[
\frac{\partial y}{\partial t} = 2 \left( t + \frac{x}{c} \right) \quad \frac{\partial^2 y}{\partial t^2} = 2
\]

\[
\frac{\partial y}{\partial x} = 2 \left( t + \frac{x}{c} \right) \frac{1}{c} \quad \frac{\partial^2 y}{\partial x^2} = 2 \frac{1}{c^2}
\]

\[
2 \frac{1}{c^2} = \frac{1}{c^2} 2 \quad \checkmark
\]

\[\therefore \quad y = \left( t + \frac{x}{c} \right)^2 \quad \text{is a solution}\]