Given: Bar of length \( L \) \( f(x=0) = 0 \) \( f(L) = 0 \) \( f'(x=L) = 0 \) \( F_x = 0 \)

Show:

a. Only odd harmonics are allowed

b. Find \( f_n \) for steel \( L = 0.5 \text{m} \)

c. \( f(x=0, t=0) = h \) \( \Rightarrow A_n = \frac{a_n}{\sqrt{n \pi n^2 \sin \frac{n\pi}{2}}} \)

d. If \( F = 500 \text{N} \) \( S = 0.00005 \text{m}^2 \) find \( A_n \)

\[
\frac{\partial^2 f}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 f}{\partial t^2} \\
\frac{c}{\sqrt{\nu}} = \sqrt{\frac{E}{\rho}}
\]

\[
\ddot{y} = \ddot{A} \cos(kx - \omega t) + \ddot{B} \sin(kx + \omega t)
\]

\[
f(x=0) = 0 \Rightarrow \ddot{A} + \ddot{B} \cos(kx + \omega t) = 0 \Rightarrow \ddot{A} = -\ddot{B}
\]

\[
\ddot{y} = \ddot{A} \cos(kx - \omega t) - \ddot{B} \sin(kx + \omega t) = -2\ddot{A} \cos(kx - \omega t)
\]

\[
\frac{\partial y}{\partial x} \bigg|_{x=L} = 0 = -2\ddot{A} \cos(\omega t + kL)
\]

\[
\cos kL = 0
\]

\[
KL = \text{odd multiples of } \frac{\pi}{2}
\]

i.e., \( kL = \frac{n\pi}{2} \) \( n = 1, 3, 5, 7, \ldots \)

\[
\boxed{R_n = \frac{n\pi}{2L}} \\
\boxed{n = 0 \text{ only only (Q.E.D.)}}
\]

\[
\omega_n = cR_n = \frac{n\pi}{2L}
\]

\[
f_n = \frac{\omega_n}{2\pi} = \frac{nC}{4L}
\]

\[
C_{\text{steel}} = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2.06 \times 10^11}{7800}} = 50.32 \text{ MPa}
\]

\[
f_n = \frac{(1)(50.32 \times 250)}{4(0.5)} = 251.4 \text{ Hz}
\]
\[
\varphi(x,0) = \frac{h}{2L} x \quad \text{and} \quad u(x,0) = 0 \implies A_0 = 0
\]

\[
\varphi(x,t) = \left( A_n \cos \omega t + B_n \sin \omega t \right) \sin k_n x \quad n = 0, \ldots, \infty
\]

\[
A_n = \frac{2}{L} \int_0^L \varphi(x,0) \sin k_n x \, dx = \frac{2}{L} \int_0^L x \sin \frac{n \pi x}{2L} \, dx
\]

\[
= \frac{2h}{n^2 \pi^2} \left[ -\frac{2x^2}{n^2 \pi^2} + \frac{4h^2 \sin \frac{n \pi x}{2L}}{n^2 \pi^2} \right]_0^L
\]

\[
= \frac{8h}{n^2 \pi^2} \sin \frac{n \pi}{2} \quad \text{Q.E.D.}
\]

\[
F = \int 5000 \, dx = 5000N + (19.5 \times 10^{10} \text{Pa}) (0.0005 \text{m}^2) \frac{h}{1.5 \text{m}}
\]

\[
h = 0.00256 \text{ m}
\]

\[
A_1 = \frac{8h}{\pi^2} = \frac{8 \times 0.00256}{\pi^2} = 0.000268 \text{ m} = 0.268 \text{ mm}
\]

\[
A_2 = 0
\]

\[
A_3 = \frac{8h}{9\pi^2} = -0.0231 \text{ mm}
\]

\[
A_4 = 0
\]

\[
A_5 = \frac{8h}{25 \pi^2} = 1.83 \mu \text{m}
\]
Given: Steel Bar
\( S = 0.0001 \ m^2 \), \( m = 15 \ kg \)
\( L = 0.25 \ m \)

Find: Fundamental Free

Node Points for Fundamental
\( u_0(x=0) \)
\( u'_0(x=1) \)

\( f \)

Free:
\( \frac{\partial^2 u}{\partial x^2} + \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{Y}{V} \sqrt{1 + f} \)

\( u = A e^{(\omega - i k)x} + B e^{(\omega + i k)x} \)

\( \frac{\partial u}{\partial x} = e^{\omega x} (i k A e^{-ix} + i k B e^{ix}) \)

\( \frac{\partial u}{\partial x} \bigg|_{x=0} = 0 = \frac{i k A}{e^{i \theta}} + \frac{i k B}{e^{-i \theta}} \)

\( A = B \)

\( f = \frac{\omega^2}{2} e^{i \omega t} (e^{-i k x} + e^{+i k x}) - 2 \frac{\omega}{k} e^{i \omega t} \cos kx \)

\( \frac{\partial^2 u}{\partial x^2} = (\omega^2) 2 \frac{\omega}{k} e^{i \omega t} \cos kx \)

\( \frac{\partial u}{\partial x} = -2 \frac{\omega}{k} e^{i \omega t} \sin kx \)

\( + YS \frac{\omega}{k} e^{i \omega t} \sin kL = -m \omega^2 \frac{\omega}{k} e^{i \omega t} \cos kL \)

\[ \tan kL = -\frac{m \omega^2}{YS} = -\frac{m \omega^2}{Y c^2 \sin k} \]

\[ = -\frac{m}{Y c^2 \sin k} = -\frac{m}{Y c^2} \]
\[
\begin{align*}
\tan kL &= - \frac{5.25\, \text{rad}}{14.25\, \text{m}} \Rightarrow (kL) = (-7.79)\, \text{rad} \\
\left( m_k = \gamma s L = (7700\, \text{kg/m}) \times (2.5\, \text{m}) \times (1000\, \text{m}^2) = 1.925\, \text{kg} \right) \\
L &= 2.11, \, 4.97, \, 51.9, \, 167.4 \quad (T=92\, \text{s}) \\
L &= 2.11 \\
L &= \frac{2.11}{2.5\, \text{m}} = \frac{8.46}{\text{m}^{-1}} \\
c &= \sqrt{\frac{\nu}{\mu}} = \sqrt{\frac{19.5\times10^4\, \text{N/m}^2}{100\, \text{kg}}} = 5032\, \text{m/s} \\
\omega_1 &= ck_1 \\
f_1 &= \frac{\omega_1}{2\pi} = \frac{ck_1}{2\pi} = \frac{(5032\, \text{m/s}) \times (8.46\, \text{m}^{-1})}{2\pi} \\
f_1 &= 16780\, \text{Hz} \\
\text{Node Points} \\
y = 0 = 2\hat{A}e^{j\omega_1 t} \cos \left(8.46\, \text{m}^{-1}\right)z \\
(z = 0.186\, \text{m}) \\
\frac{y_1(z=0)}{y_1(z=L)} &= \frac{2\hat{A}e^{j\omega_1 t}}{2\hat{A}e^{j\omega_1 t} \cos kL} = \frac{1}{\cos \left(8.46\, \text{m}^{-1}\right) \times (1.25\, \text{m})} \\
&= \frac{1.800}{2.12} \quad (\text{Out of Phase}) \\
L &= \frac{k_2 (12.5\, \text{m}) = 4.945 \quad (L = 19.86\, \text{m}^{-1})} \\
f_2 &= \frac{c_k^2}{2\pi} = 15900\, \text{Hz}
3.9.1. Given: \( y = \left[ A \cosh x + B \sinh x + C \cos gx + D \sin gx \right] \cos (x) \)

\[
\frac{\partial^2 y}{\partial t^2} = - \frac{K^2 c^2}{2} \frac{\partial^2 y}{\partial x^2} \\
\Rightarrow c^2 = \frac{Y}{\mu}
\]

\[
\frac{\partial y}{\partial t} = \left[ - \omega \sin (\omega t + \phi) \right] \\
\frac{\partial^2 y}{\partial t^2} = \left[ - \omega^2 \cos (\omega t + \phi) \right]
\]

\[
\frac{\partial y}{\partial x} = \left[ A \cosh x + B \sinh x - C \sin gjx + D \cos gjx \right] \cos (x)
\]

\[
\frac{\partial^2 y}{\partial x^2} = \left[ A \cosh^2 x + B \sinh^2 x - C \cos gjx - D \sin gjx \right] \cos (x)
\]

\[
\frac{\partial^3 y}{\partial x^3} = \left[ A \cosh^3 x + B \sinh^3 x + C \cosh gjx - D \sin gjx \right] \cos (x)
\]

\[
\frac{\partial^4 y}{\partial x^4} = \left[ A \cosh^4 x + B \sinh^4 x + C \cosh gjx + D \sin gjx \right] \cos (x)
\]

\[- \omega^2 \left[ A \cosh x + B \sinh x + C \cos gx + D \sin gx \right] \cos (x) = - \frac{K^2 c^2}{2} \left[ A \cosh x + B \sinh x + C \cos gx + D \sin gx \right] \cos (x)
\]

\[+ \omega^2 = + \frac{K^2 c^2}{2} g \]

\[
g^4 = \frac{Y}{K c^2} \left( \frac{\omega^2}{\omega^2} \right) = \frac{\omega^4}{\nu^4} \\
g = \frac{\omega}{\nu}
\]
3.13.1 Torsional Mode

Given: \( L = 100 \text{ cm} \quad d = 1 \text{ cm} \quad A_l \)

Find: \( \tau (\theta = 360^\circ) \), \( c \), \( \text{Lowest Freq} \)

\[ \tau = \frac{G}{2} \frac{d^4}{dx^4} \frac{d\phi}{dx} \]
\[ = \left(2.4 \times 10^{10} \frac{N}{m^2}\right) \frac{1}{2} \pi \left(0.01 m\right)^4 \frac{2\pi}{1 m} \text{m} \]
\[ = 148 \text{ N-m} \]

\[ c = \sqrt{\frac{GD}{\rho}} = \sqrt{\frac{2.4 \times 10^{10} \frac{N}{m^2}}{2700 \text{ kg/m}^3}} = 2980 \text{ m/s} \]

\[ f = \frac{nc}{2L} \]
\[ = \frac{2980 \text{ m/s}}{2 \text{ cm}} = 1490 \text{ Hz} \]