Maxwell's Equations are the four equations that (together with $\vec{F} = q(\vec{E}+\vec{v}\times\vec{B})$) form the basis of all of classical electromagnetism. Taken together, these laws also explain what light is: a traveling electromagntic wave. They are:

$$\oint_S \vec{E}\cdot\hat{n}\,dA = \frac{Q_{\text{inside}}}{\epsilon_0} \qquad \oint_C \vec{E}\cdot d\vec{l} = -\frac{d\phi_{\text{m}}}{dt}$$

$$\oint_S \vec{B}\cdot\hat{n}\,dA = 0 \qquad \oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_C + \mu_0\epsilon_0\frac{d\phi_{\text{e}}}{dt}$$

These equations are:

- Gauss's Law: says that electric charges create electric fields
- Faraday's Law: says that time-varying magnetic fields create electric fields
- Gauss's Law for Magnetism: says that magnetic monopoles do not exist
- Generalized form of Ampere's Law (the Ampere-Maxwell Law): Says that both currents and time-varying electric fields will create magnetic fields

Most of these are not new. The truly new addition is the extra $+\mu_0\epsilon_0\frac{d\phi_{\text{e}}}{dt}$ added to the end of Ampere's Law. This means that a time varying electric flux (such as inside a capacitor that is in the process of being charged) will create a magnetic field. We can define the displacement current as $I_{\text{d}} = \epsilon_0\frac{d\phi_{\text{e}}}{dt}$ (where this current points in the same direction as $\frac{d\vec{E}}{dt}$) and then use this as a current in Ampere's Law.

As we have done before, the Ampere-Maxwell equation is useful in cases of cylindrical symmetry. In that case our Amperian loop is a circle of radius r centered on the axis of symmetry and which passes through the point in space where we are attempting to find the magnetic field (this point is space is clearly a distance r away from the symmetry axis). For this symmetry the Ampere-Maxwell equation reduces to

$$B2\pi r = \mu_0(I + I_{\text{d}})_{\text{enclosed}}$$

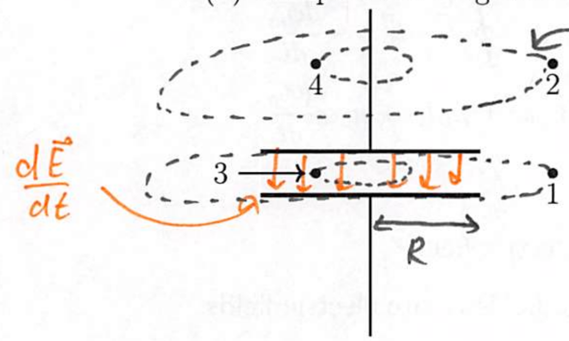
where the right hand side has both the real current and the displacement current that are enclosed by our Amperian loop.

If you wish to determine the direction of the induced magnetic field, you can say that the displacement current points in the direction of $\frac{d\vec{E}}{dt}$. You can use a right hand rule for finding the field direction.

Below are 4 positions around a parallel plate capacitor. The capacitor is being charged. This means that current is flowing through the wires, and that the electric field inside the capacitor is changing in time.

(a) Compare the magnetic fields at position 1 and position 2. Which is bigger?

(b) Compare the magnetic fields at position 3 and position 4. Which is bigger?



a) $\boxed{B_1 = B_2}$ same r away

#2 loop contains current I

#1 loop contains displacement current I_d

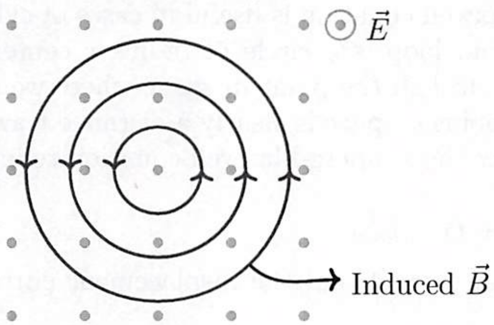
$I_d = \varepsilon_0 \frac{dE}{dt} \pi R^2 = I$

b) $\boxed{B_4 > B_3}$ same r away

#4 loop contains the current

#3 loop encloses some of the displacement current

Inside a circular capacitor the electric field is pointing out of the page. There is a magnetic field that makes counter-clockwise circles. Is the electric field increasing or decreasing in strength?



A ccw B-field means that the displacement current points out of the page.

The displacement current points along the CHANGE in the electric field with time.

The E-field is changing in the same direction it points.

Therefore E IS INCREASING.