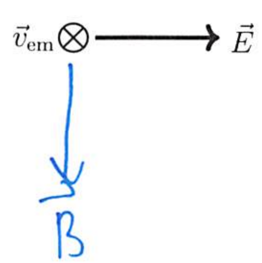
We have seen from Maxwell's equations that a time-varying magnetic field induces an electric field (Faraday's Law) and that a time-varying electric field induces a magnetic field (Ampere-Maxwell Law). Therefore there is a self-consistent solution that allows for electric and magnetic fields in a region of space with no charges: an electromagnetic wave features a traveling wave of time-varying electric and magnetic fields that mutually create each other. One solution is:

$$E_y = E_0 \sin(kx - \omega t) \qquad\qquad B_z = B_0 \sin(kx - \omega t)$$

These are traveling waves, moving in the positive x direction with speed $v = \frac{\omega}{k} = \lambda f$. The electric field oscillates along the y direction with amplitude E_0 while the magnetic field oscillates (in phase with the electric field) along the z direction with amplitude B_0. ω is the angular frequency of the wave (units of rad/s), while $f = \frac{\omega}{2\pi}$ is the frequency of the wave (units of Hz). k is the angular wave number of the wave (units of rad/m) while $\lambda = \frac{2\pi}{k}$ is the wavelength of the wave (units of m).

In order for these solutions to satisfy Maxwell's equations a few things need to be true.

- The electric and magnetic fields oscillate in phase: these fields will both be at their maxima at the same place and time and will both equal 0 at the same place and time.
- All EM waves in a vacuum travel at the speed of light; $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8$ m/s. Therefore $c = \lambda f = \frac{\omega}{k}$.
- The wave travels in the $\vec{E} \times \vec{B}$ direction. In our chosen definition, $\vec{E}$ oscillates along y, $\vec{B}$ oscillates along z, and the wave travels along $+x$. It is true that $\hat{j} \times \hat{k} = +\hat{i}$.
- The electric and magnetic field amplitudes are related as $E_0 = cB_0$. In fact, because the fields are in phase it will be true that $E = cB$ is generally true for any position and time.



An electromagnetic wave is moving into the page. At some place/time the electric field for this wave points to the right. What is the direction of the magnetic field of the wave at this moment in space and time?

$\vec{V}$ is in the $\vec{E} \times \vec{B}$ direction

The electric field of an em wave traveling in a vacuum is given by

$$E_y = (800\ \text{N/C})\sin\left[kx - (3.5\times 10^{15}\ \text{rad/s})t\right]$$

(a) What is B_0 (the magnetic field amplitude)?

$$B_0 = \frac{E_0}{c} = \frac{800\ \text{N/C}}{3.0\times10^{8}\ \text{m/s}} = 2.669\times10^{-6}\ \text{T}$$

$$\boxed{B_0 = 2.67\ \mu\text{T}}$$

(b) What is k (the angular wave number)?

$$V = \frac{\omega}{k} \quad \text{so} \quad k = \frac{\omega}{c} = \frac{3.5\times10^{15}\ \text{rad/s}}{3.00\times10^{8}\ \text{m/s}}$$

$$\boxed{k = 1.17\times10^{7}\ \frac{\text{rad}}{\text{m}}}$$

(c) What is f (its frequency)?

$$f = \frac{\omega}{2\pi} = \frac{3.5\times10^{15}\ \frac{\text{rad}}{\text{s}}}{2\pi\ \text{rad/cycle}} = 5.57\times10^{14}\ \text{Hz}$$

$$\boxed{557\ \text{THz}}$$

(d) What is λ (its wavelength)?

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.17\times10^{7}\ \frac{\text{rad}}{\text{m}}} = 5.37\times10^{-7}\ \text{m}$$

$$\boxed{\lambda = 537\ \text{nm}}$$