We previously saw that an electromagnetic wave moving in the $+x$ direction could be described by

$$E_y = E_0 \sin(kx - \omega t) \qquad B_z = B_0 \sin(kx - \omega t)$$

Electromagnetic waves carry energy. We already know the energy densities of both the electric and magnetic fields, so it is unsurprising that the energy density of an electromagnetic wave will simply be the sum of them:

$$u = u_e + u_m = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2.$$

But, for any point in space and moment in time, the electric and magnetic field strengths in the wave are related as $E = cB$, where $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ is the speed of light. But from this we can see that $= u_e = u_m$: exactly half of the energy in an em wave is stored in its electric field and exactly half in its magnetic field. Therefore

$$u = u_e + u_m = \epsilon_0 E^2 = \frac{B^2}{\mu_0} = \frac{EB}{\mu_0 c}.$$

We can define the Poynting vector as $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$. This is flow of power due to the wave. It points in the direction of energy transfer (the direction of the wave's travel) and represents the power (energy per time) delivered per unit area by the wave. It has units of W/m^2.

The intensity of light is the average power per area delivered by the wave. We see that

$$I = |\vec{S}|_{av} = \frac{E_0 B_0}{2\mu_0}$$

where the factor of $\frac{1}{2}$ comes from the fact that we are time averaging (i.e. the average value of $\sin^2(kx - \omega t)$ is $\frac{1}{2}$).

If a source radiates energy as a point source the the intensity at a distance r away will simply by $I = \frac{P_{av}}{4\pi r^2}$ where P_{av} is the average power provided by the source.

A light bulb gives off em waves with a 60 W power. It radiates like a point source. At a distance of 2.0 m from the light bulb...

(a) What is the intensity of the light?

$$I = \frac{P_{av}}{4\pi r^2} = \frac{60\,W}{4\pi(2.0\,m)^2} = 1.19\,\frac{W}{m^2}$$

(b) What is the electric field amplitude?

$$I = \frac{E_0 B_0}{2\mu_0} \qquad B_0 = \frac{E_0}{c} \qquad \text{so} \quad I = \frac{E_0^2}{2\mu_0 c}$$

$$E_0 = \sqrt{2\mu_0 c I} = \sqrt{2(4\pi\times10^{-7}\,\tfrac{T\cdot m}{A})(3.0\times10^8\,m/s)(1.19\,W/m^2)} = 30.0\,\frac{N}{C}$$

(c) What is the magnetic field amplitude?

$$B_0 = \frac{E_0}{c} = \frac{30.0\,N/C}{3.0\times10^8\,m/s} = 1.00\times10^{-7}\,T$$

or

$$0.100\,\mu T$$

At this point in space and at a moment in time when the fields are at their amplitudes...

(d) What is the energy density of the wave?

$$u = \frac{EB}{\mu_0 c} \qquad \frac{(30\,N/C)(1.0\times10^{-7}\,T)}{\mu_0 c} = 7.96\times10^{-9}\,J/m^3$$

(e) What is the instantaneous magnitude of the Poynting vector?

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} \qquad S = \frac{EB\sin 90^\circ}{\mu_0} = uc$$

$$(7.96\times10^{-9}\,J/m^3)(3\times10^8\,m/s) = 2.39\,\frac{W}{m^2}$$