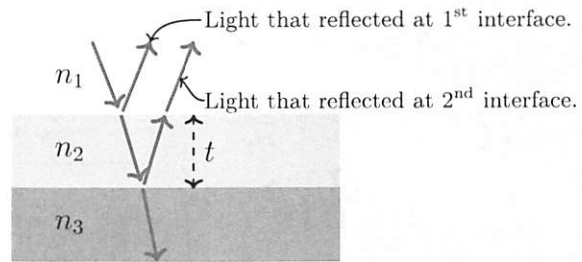


We have already discussed the fact that when light goes from one material into another its speed will change. We also know that the speed of a wave is given by  $v = \lambda f$ . Thus a wave's wavelength and/or frequency must change. It turns out that a wave's wavelength will change when it moves from one material into a material with a different index of refraction; its frequency will not change. We could write this as  $\lambda_n = \frac{\lambda}{n}$  where  $\lambda_n$  is the wavelength inside the material with index  $n$  while  $\lambda$  is its wavelength in vacuum (or air, which has an index that is practically equal to 1).

We now consider thin film interference. Three different materials are layered together. The middle material, with index  $n_2$ , has a thickness  $t$ . We will always assume that the light traveling through material #1 (with index  $n_1$ ) is near normally incident ( $\theta \approx 0^\circ$ ). Some light reflects off of the first interface. Some light passes through this interface then reflects off of the second interface



(where materials #2 and #3 meet) then refracts back into material #1. These two rays of light will interfere. The path length difference between the two rays will be  $2t$ , because the light that refracted at the first surface will travel an extra distance  $t$  going down and then another  $t$  going up. This path length difference will need to be compared to  $\frac{\lambda}{n_2}$ , which is the wavelength of the light inside the thin film.

The other relevant fact is that when light traveling in one medium strikes the surface of a second medium in which light travels slower (has a larger index of refraction), there is a  $180^\circ$  phase shift in the reflected light. A  $180^\circ$  phase shift at a reflection is equivalent to a path length difference of  $\lambda/2$ . There will be no phase shifts when  $n_1 > n_2 > n_3$ . There will be a phase shift at both reflections when  $n_3 > n_2 > n_1$ . There will be a phase shift at only the first reflection when  $n_2$  is the largest index. There will be a phase shift at only the second reflection when  $n_2$  is the smallest index.

We can put this together and we get ( $m = 1, 2, 3, \dots$ ):

$$2t = m \frac{\lambda}{n_2}$$

The path length difference is an integer number of wavelengths. This will give constructive interference whenever there is a  $180^\circ$  phase shift at either both or neither of the two reflections. This will give destructive interference whenever there is a  $180^\circ$  phase shift at exactly one of the two reflections.

$$2t = \left(m - \frac{1}{2}\right) \frac{\lambda}{n_2}$$

The path length difference is an integer plus a half number of wavelengths. This will give destructive interference whenever there is a  $180^\circ$  phase shift at either both or neither of the two reflections. This will give constructive interference whenever there is a  $180^\circ$  phase shift at exactly one of the two reflections.

A bubble of soap water ( $n = 1.33$ ) is surrounded on both sides by air. It is found that 680 nm is the longest wavelength of light that will interfere constructively in the reflection. What is thickness of this film? What are the two longest wavelengths that will interfere destructively in the reflection?

$n_2 = 1.33$     $n_1 = n_3 = 1.00$     $180^\circ$  phase shift at 1<sup>st</sup> reflection only,

$$2t = \left(m - \frac{1}{2}\right) \frac{\lambda}{n_2} \quad \text{for constructive}$$

$$2t = \frac{1}{2} \frac{680 \text{ nm}}{1.33}$$

$$t = 127.8 \text{ nm}$$

$$\text{Destructive: } 2t = m \frac{\lambda}{n_2} \quad \lambda = \frac{2tn_2}{m}$$

$$\text{two longest: } \frac{2(127.8 \text{ nm})(1.33)}{1} = 340 \text{ nm} \quad \frac{2tn_2}{2} = \underline{170 \text{ nm}}$$

A 200 nm thick anti-reflective coating with an index of  $n = 1.45$  is applied to a pair of eyeglasses. (The glasses are made of a material with  $n = 1.55$ ) What is the longest wavelength that will interfere destructively in the reflection? What is the longest wavelength that will interfere constructively in the reflection?

$180^\circ$  shift at both reflections

$$\text{Destructive: } 2t = \left(m - \frac{1}{2}\right) \frac{\lambda}{n_2}$$

$$\lambda = \frac{2tn_2}{\frac{1}{2}} = 4(200 \text{ nm})(1.45) = \underline{1160 \text{ nm}}$$

$$\text{Constructive: } 2t = m \frac{\lambda}{n_2}$$

$$\lambda = \frac{2tn_2}{m} = \underline{580 \text{ nm}}$$