The Biot-Savart Law

SP212 - General Physics II

United States Naval Academy
Annapolis

Earlier this semester we learned that point charges create electric fields ($\vec{E} = k\frac{q}{r^2}\hat{r}$) and that point charge feel electric fields ($\vec{F} = q\vec{E}$).

We have just learned that moving point charges will feel a force from a magnetic field: $\vec{F} = q\vec{v} \times \vec{B}$.

It should not surprise us that moving charges likewise create magnetic fields.

The permeability constant

μ_0 (pronounced mu-nought) is a constant known as the permeability of free space. It has a value of $\mu_0 = 4\pi \times 10^{-7}$ T $\cdot$ m/A. We will find that μ_0 appears in many magnetic equations in a way similar to how $\frac{1}{\epsilon_0}$ appear in many electrostatics equations.

Interestingly, after the SI redefinition in 2019 μ_0 is no longer an exactly defined quantity. Instead it is now a measured quantity. But its current measured value is remarkably close to what is given above. Our textbook uses the value above, so we will as well.

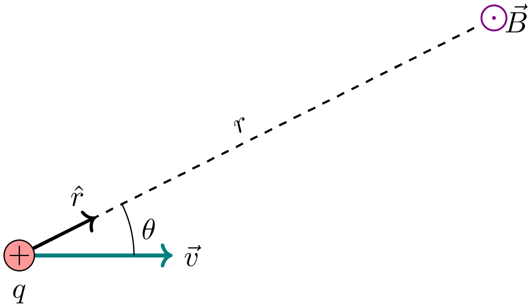
The magnetic field created by a mving point charge can be determined by the Biot-Savart law.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

Biot-Savart Law

- $\vec{B}$ is the magnetic field [units are tesla, T]
- q is the charge [units are coulombs, C]
- $\vec{v}$ is the velocity of the point charge [units are meters per second, m/s]
- $\vec{r}$ is a vector that points from the point charge to the position in space where we are determining the magnetic field.
    - r is the distance between the point charge and the position in space where we are determining the magnetic field [units are meters, m]
    - $\hat{r}$ is a unit vector (size 1) that points from the point charge to the position in space where we are determining the magnetic field.
- $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ is the permeability of free space

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$ Biot-Savart Law



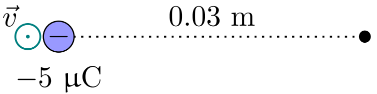
- Magnitude: $B = \dfrac{\mu_0}{4\pi} \dfrac{|q|v\sin\theta}{r^2}$
- Right hand rule: point fingers in the direction of $q\vec{v}$ (along $\vec{v}$ for $q > 0$, opposite $\vec{v}$ for $q < 0$). Curl fingers along $\hat{r}$. Thumb gives the direction of $\vec{B}$.

A subtlety: the vector $\vec{r}$ points from the moving point charge to the position is space where we are determining the magnetic field. But the charge is moving so it passes through that point in space: it is only *there* for one brief instant. So this equation is valid for only one brief instant. When?

Magnetic fields carry information at the speed of light. So this equation gives us the magnetic field at the tip of $\vec{r}$ at the moment in time r/c later than the moment in time when the point charge was located at the base of $\vec{r}$. (c is the speed of light.) But the speed of light is very fast, so in this course we will treat this as instantaneous.

Example:

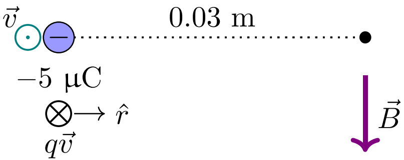
A $-5\ \mu C$ point charge is moving out of the page at 6000 m/s. What is the magnetic field due to this moving point charge 0.03 m to its right?



Solution:

The magnitude of the magnetic field is $\frac{\mu_0}{4\pi} \frac{(5 \times 10^{-6}\ \text{C})(6000\ \text{m/s}) \sin 90^\circ}{(0.03\ \text{m})^2} = 3.33 \times 10^{-6}\ \text{T}.$

$q\vec{v}$ points into the page: opposite $\vec{v}$ because q is negative. $\hat{r}$ points to the right. By the RHR, $\vec{B}$ points down.



The magnetic field is 3.33 μT pointing down.