Energy in EM Waves

SP212 - General Physics II

UNITED STATES
NAVAL ACADEMY
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$E_y(x,t) = E_0 \sin(kx - \omega t) \qquad B_z(x,t) = B_0 \sin(kx - \omega t)$ Travelling waves

$v_{\text{em}} = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s} = \lambda f = \frac{\omega}{k}$ Speed of light

$E_0 = cB_0$ Amplitude relation

$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$ Angular variables

- E_0 is the electric field amplitude [units are newtons per coulomb, N/C]
- B_0 is the magnetic field amplitude [units are tesla, T]
- k is the angular wavenumber [units are radians per meter, rad/m]
- ω is the angular frequency [units are radians per second, rad/s]
- λ is the wavelength [units are meters, m]
- f is the frequency [units are hertz, Hz]

Energy in an EM wave

Energy density of an electric field: $u_{\mathrm{e}} = \dfrac{1}{2}\epsilon_0 E^2$

Energy density of a magnetic field: $u_{\mathrm{m}} = \dfrac{B^2}{2\mu_0}$

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$E = cB$ at at position/time on an EM wave, and $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

This gives $u = \epsilon_0 E^2 = \frac{B^2}{\mu_0} = \frac{EB}{\mu_0 c}$ as the energy density of an EM wave

Energy Density

$$u = \frac{EB}{c\mu_0}$$

Energy density of an EM wave

The Poynting vector is the rate of energy transfer by an EM wave per unit area on the wave front.

- u is the energy density [units are joules per cubic meter, $\mathrm{J/m^3}$]
- $\mu_0 = 1.26 \times 10^{-6}$ $\mathrm{T \cdot m/A}$ is permeability constant
- $c = 3.00 \times 10^8$ m/s is the speed of light
- E is the electric field strength [units are newtons per coulomb, N/C]
- B is the magnetic field strength [units are tesla, T]

This is true for a single moment in time, and for a volume small enough that E and B are relatively constant.
The EM wave moves, so this energy is carried in the direction of the wave's travel.

Poynting Vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Poynting vector

The Poynting vector is the rate of energy transfer by an EM wave per unit area on the wave front.

- $\vec{S}$ is the Poynting vector [units are watts per square meter, $\mathrm{W/m^2}$]
- $\mu_0 = 1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}$ is permeability constant
- $\vec{E}$ is the electric field [units are newtons per coulomb, N/C]
- $\vec{B}$ is the magnetic field [units are tesla, T]

- $\vec{S}$ points in the direction of the wave's travel (energy moves along with the wave)
- Multiplying S by an area perpendicular to the direction of the wave gives power - the energy carried per time.

Intensity

Intensity is the energy carried per time per area by an electromagnetic wave.

$$I = S_{\text{avg}} = \frac{1}{2}\frac{1}{\mu_0}E_0 B_0 = \frac{1}{2c\mu_0}E_0^2$$

Intensity

- I is the intensity of the EM wave [units are watts per square meter, W/m^2]
- S_{avg} is the time-average value of the magnitude of the Poynting vector [units are Watts per square meter, W/m^2]
- $c = 3.00 \times 10^8$ m/s is the speed of light in a vacuum
- $\mu_0 = 1.26 \times 10^{-6}\ \text{T} \cdot \text{m/A}$ is permeability constant
- E_0 is the electric field amplitude [units are newtons per coulomb, N/C]
- B_0 is the magnetic field amplitude [units are tesla, T]

Intensity of a point source

Intensity is the energy carried per time per area by an electromagnetic wave.

$$I = \frac{P_{\text{source}}}{4\pi r^2}$$

Intensity of a point source

- I is the intensity of the EM wave [units are watts per square meter, $\mathrm{W/m^2}$]
- P_{source} is the power emitted by the point source [units are watts, W]
- r is the distance from the point source to the position in space where we are determining the EM wave intensity [units are meters, m]

A point source is a source that emits EM wave evenly in all directions.

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$$B_0 = \frac{E_0}{c} = \frac{17.34\ \text{N/C}}{3.00\times 10^8\ \text{m/s}} = 5.78\times 10^{-8}\ \text{T} = 57.8\ \text{nT}$$