

Images from Thin Lenses

SP212 - General Physics II



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The primary equations describing images created by thin lenses are exactly the same as the ones that described images created by spherical mirrors.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Image Equation

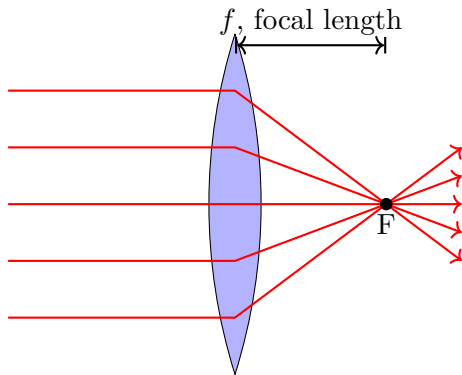
$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Magnification

The biggest difference lies in sign conventions, but we can always use one rule: the positive side is where the light goes.

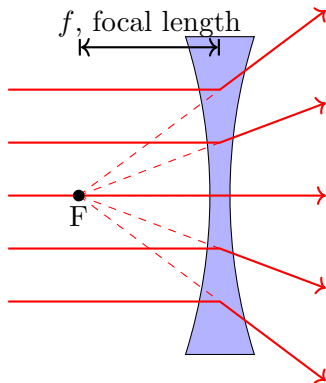
- Mirrors reflect light. Therefore the positive side is the same side of the mirror as the object.
- Lenses transmit light. Therefore the positive side is the opposite side of the lens from the object.

A converging lens is thickest in the middle and thinner at the top and bottom: like a cat's eye shape.



Parallel rays of light converge at the focal point. The focal length of a converging lens is positive.

A diverging lens is thinnest in the middle and thicker at the top and bottom: like an hourglass shape.



Parallel rays of light diverge in a way that they trace back to the focal point. The focal length of a diverging lens is negative.

There are several special rays of light whose behavior is easy to predict.

Converging lenses

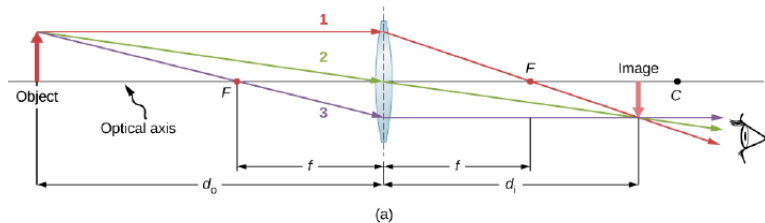


Fig. 2.21, *OpenStax University Physics Vol. 3*

- 1 A ray parallel to the optical axis passes through the focal point on the other side of the lens.
- 2 A ray passing through the center of the lens passes through in a straight line.
- 3 A ray passing through the focal point exits the lens parallel to the optical axis.

Diverging lenses

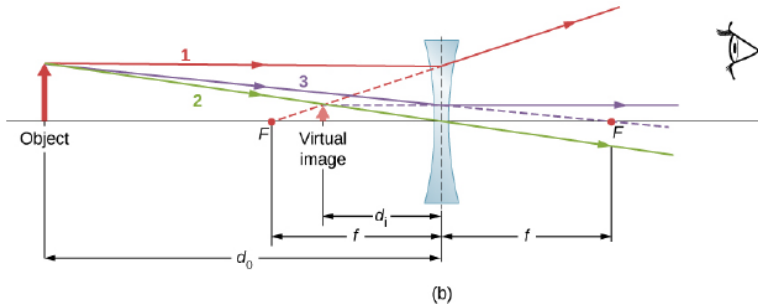


Fig. 2.21, *OpenStax University Physics Vol. 3*

- 1 A ray parallel to the optical axis exits along the line that passes through the focal point on the same side of the lens.
- 2 A ray passing through the center of the lens passes through in a straight line.
- 3 A ray that enters the lens along the line that passes through the focal point on the other side of the lens exits parallel to the optical axis.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Image Equation

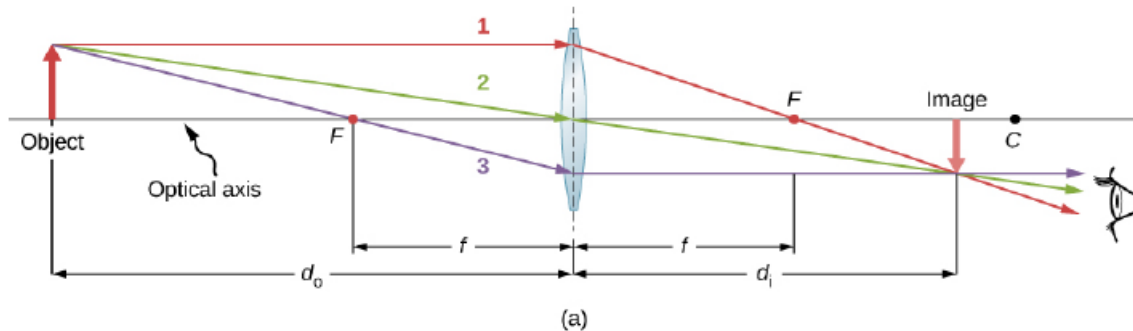
- d_o is the object distance (distance between the object and the lens)
 - ▶ For a single lens this will always be positive.
- d_i is the image distance (distance between the image and the lens)
 - ▶ d_i is positive for real images.
 - ▶ d_i is negative for virtual images.
- f is the focal length of the lens
 - ▶ f is positive for a converging lens.
 - ▶ f is negative for a diverging lens.
- A real image has a positive image distance. Rays of light actually converge at the image.
 - ▶ A converging lens creates a real image when $d_o > f$.
- A virtual image has a negative image distance. Rays of light trace back to the image.
 - ▶ A converging lens creates a virtual image when $d_o < f$.
 - ▶ A single diverging lens can only create virtual images.

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Magnification

- Magnification is the ratio between the image size and the object size: $m = \frac{h_i}{h_o}$
 - ▶ When $m > 0$ the image is upright.
 - ▶ When $m < 0$ the image is inverted (upside down).
 - ▶ When $|m| > 1$ the image is enlarged (bigger than the object).
 - ▶ When $|m| < 1$ the image is shrunken (smaller than the object).
- Magnification can be expressed as $m = -\frac{d_i}{d_o}$
- Real images have a positive d_i and virtual images have a negative d_i
 - ▶ For a single lens, real images are always inverted.
 - ▶ For a single lens, virtual images are always upright.

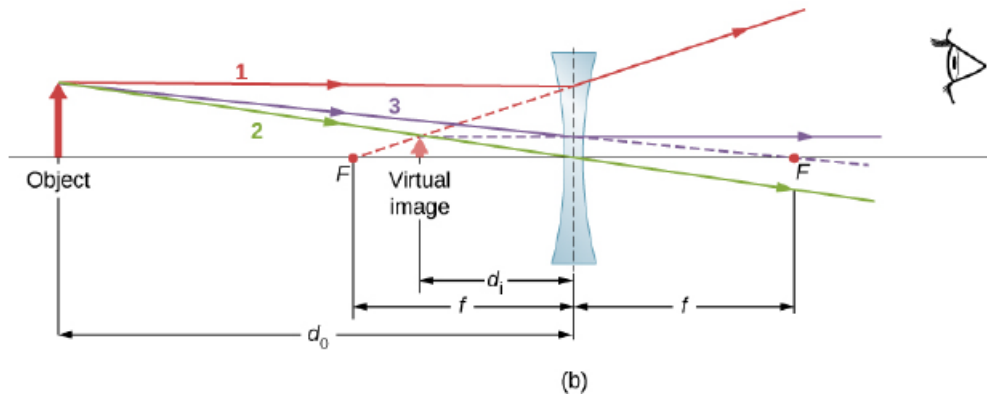
Your eye traces back the various rays of light to the image position.



A real image is formed when the rays of light actually converge at the image position. This image could be projected onto a screen or used to develop film.

The image is on the side of the lens opposite the object, so d_i is positive (this is the side “where the light goes”).

Your eye traces back the various rays of light to the image position.



A virtual image is formed when the rays of light don't actually converge at the image position; you need to trace the rays back to the convergence.

The image is on the same side of the lens as the object, so d_i is negative.

Example:

An object is placed 20 cm from a lens with $f = -10$ cm. Where is the image? What is its magnification?

$$\frac{1}{20 \text{ cm}} + \frac{1}{d_i} = \frac{1}{-10 \text{ cm}} \quad \text{so } d_i = -6.67 \text{ cm}$$

The image is 6.67 cm from the lens, on the same side as the object (virtual image).

$m = -\frac{-6.67 \text{ cm}}{20 \text{ cm}} = +0.333$ is the magnification. The image is one-third the size of the object. The image is upright.

The lensmaker's equation

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

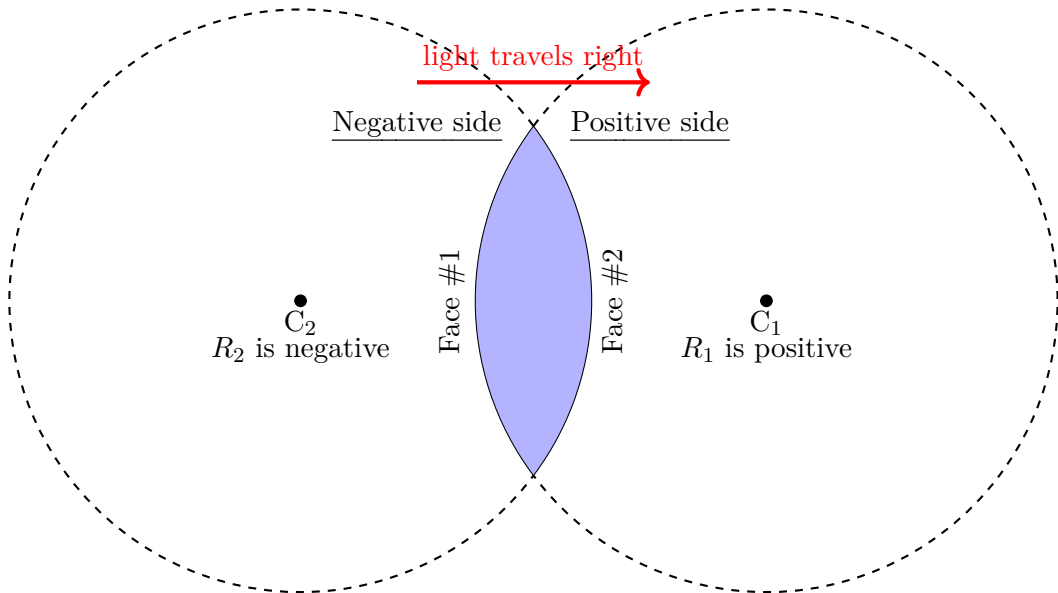
The lensmaker's equation

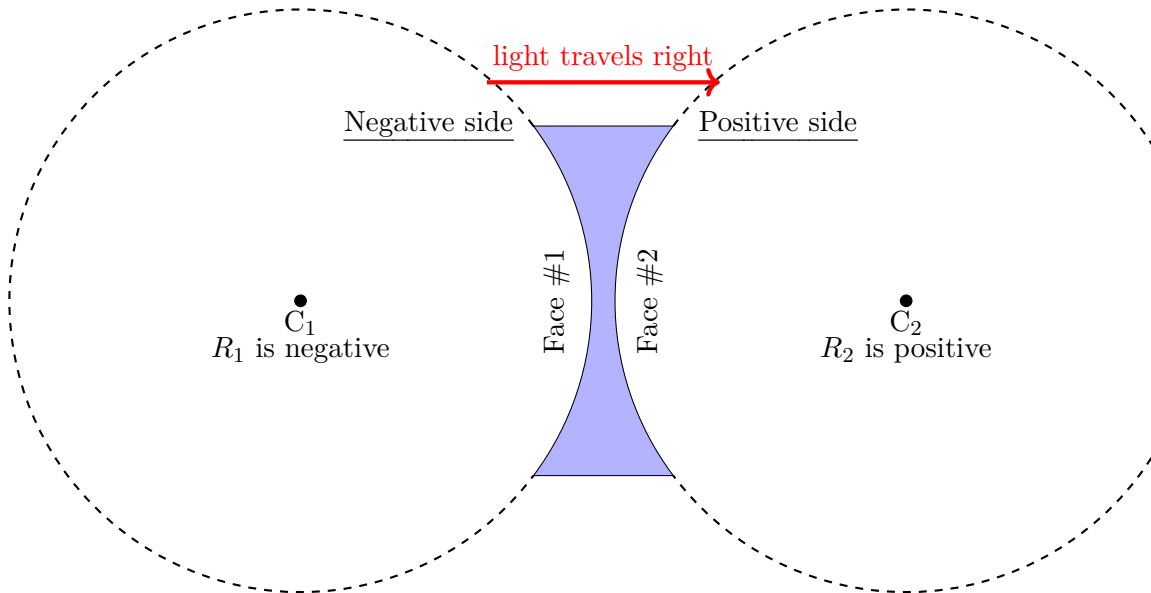
- f is the focal length of the lens
- n is the index of refraction of the material the lens is made out of
- R_1 is the radius of curvature of the first side of the lens
- R_2 is the radius of curvature of the second side of the lens

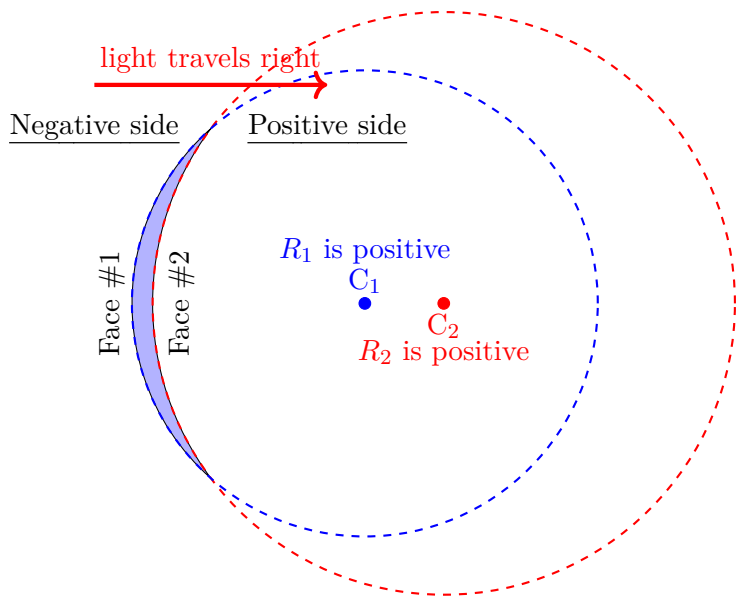
This equation assumes that the lens is surrounded by air or vacuum.

Sign convention: Pick a direction that the light is moving (say left to right). R_1 is the radius of the side the light hits first (the left side if the light is moving left to right).

The positive side is where the light goes (the right side if the light is moving left to right). If a center of curvature is on that side then it is positive. If it is on the other side it is negative.

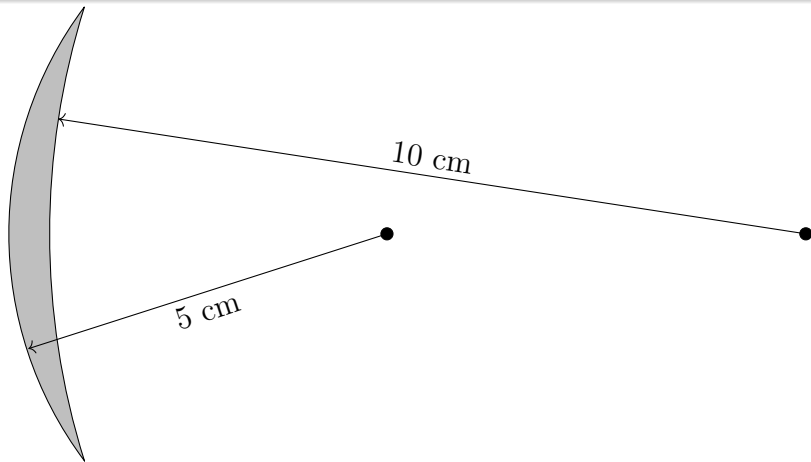




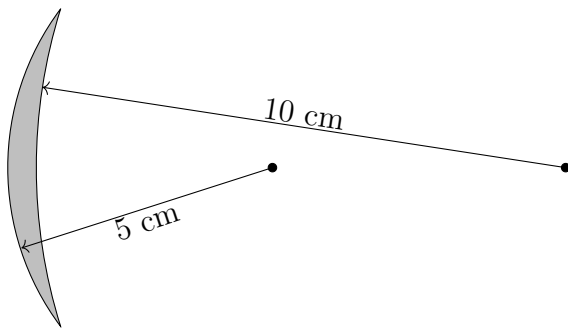


Example:

The lens shown below is made of a material with an index of $n = 1.4$. What is its focal length (in air)?



I'll assume that the light moves left to right. Both centers of curvature are on the right (positive) side. The light first hits the side with $|R| = 5$ cm, so $R_1 = +5$ cm and $R_2 = +10$ cm.



$$\frac{1}{f} = (1.4 - 1) \left(\frac{1}{5 \text{ cm}} - \frac{1}{10 \text{ cm}} \right)$$

$$f = +25 \text{ cm}$$