Magnetic Field due to Current Loops

SP212 - General Physics II

UNITED STATES
NAVAL ACADEMY
ANNAPOLIS

Moving charges feel a magnetic field: $\vec{F} = q\vec{v} \times \vec{B}$

Moving charges create magnetic fields: $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{q\vec{v} \times \hat{r}}{r^2}$

A current-carrying wire contains moving charges. So a current creates a magnetic field. A steady current creates a steady magnetic field.

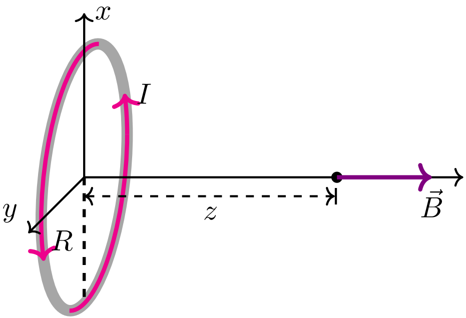
Biot-Savart law for a current element: $d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{I\, d\vec{l} \times \hat{r}}{r^2}$

$d\vec{B}$ is the differential (very small) magnetic field created by a small current element $I\, d\vec{l}$ that has length dl and points in the direction of $d\vec{l}$.

$$B_{\text{loop}} = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}}$$

On-axis B-field due to a circular current-carrying wire loop

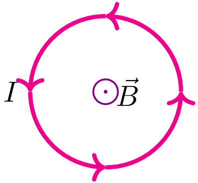
- B_{loop} is the magnetic field strength [units are tesla, T]
- I is the current in the wire loop [units are amperes, A]
- R is the radius of the circular loop [units are meters , m]
- z is the on-axis distance from the center of the loop [units are meters , m]

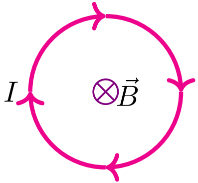


As a special case of the above equation, at the center of the circle $z = 0$ and the equation reduces to $B_{\text{center}} = \frac{\mu_0 I}{2R}$.

Sometimes, you might have a coil, in which the wire is wrapped around the circle N times. In this case the magnetic field is just multiplied by this factor N.

Right Hand Rule: Curl the fingers of your right hand in the direction of the current that is flowing through this circular loop. You thumb will give the direction of the magnetic field along this axis.

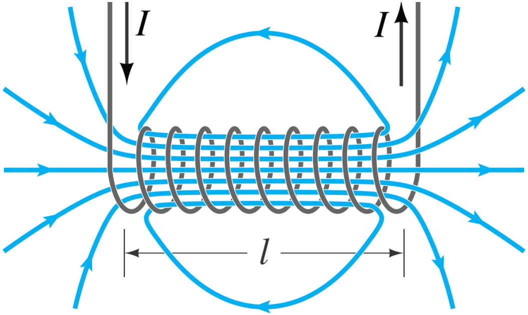




- A counter-clockwise current creates a magnetic field at its center that points out of the page.
- A clockwise current creates a magnetic field at its center that points into the page.

Solenoids

Another important example is that of the solenoid, in which the current-carrying wire wrapped into a helical coil.



https://commons.wikimedia.org/wiki/File:Solenoid_and_Ampere_Law.png

Solenoids are particularly useful in that they will create a very uniform magnetic field inside them.

$$B_{\text{solenoid}} = \mu_0 n I = \mu_0 \frac{N}{l} I$$

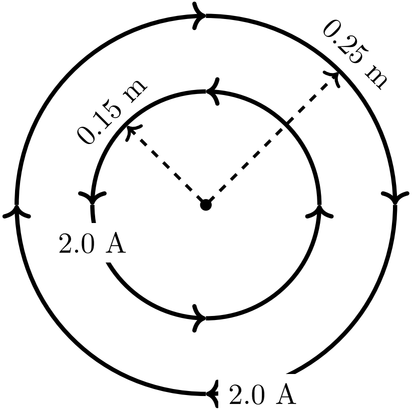
Magnetic field inside an ideal solenoid

- B_{solenoid} is the magnetic field strength [units are tesla, T]
- n is the turn density (turns per length) of the wire wrapped around the solenoid. [units are turns per meter]. $n = \frac{N}{l}$ where N is the total number of turns and $l4$ is its length.
- I is the current flowing through the solenoid [units are amperes, A]

The magnetic field <u>outside</u> the solenoid is zero.

This equation is strictly valid for an ideal solenoid: infinitely long and with the wrapping being perfectly tight. The equation is pretty valid for many real solenoids as long as they are a good bit longer than their diameter and wrapped pretty tightly.

Example:



A wire loop of radius 0.15 m carries a counter-clockwise current of 2.0 A; it is concentric with another loop of radius 0.25 m that carries a 2.0 A current clockwise. (a) What is the net magnetic field (size and direction) at the center of the two loops? (b) What is the net magnetic field (size and direction) at a point 0.30 m above their center?

Solution:

(a) What is the net magnetic field (size and direction) at the center of the two loops?

At the center $z = 0$ and $B = \dfrac{\mu_0 I}{2R}$. The small loop creates a field $\dfrac{\mu_0(2.0 \text{ A})}{2(0.15 \text{ m})} = 8.38 \times 10^{-6}$ T. By the RHR it points out of the page.

The large loop creates a field $\dfrac{\mu_0(2.0 \text{ A})}{2(0.25 \text{ m})} = 5.03 \times 10^{-6}$ T. By the RHR it points into the page.

The net magnetic field at the center is 3.35×10^{-6} T pointing out of the page.

Solution:

(b) What is the net magnetic field (size and direction) at a point 0.30 m above their center?

$B_{\text{loop}} = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}}$ and this is at a height of $z = 0.30$ m.

The small loop creates a field $\frac{\mu_0 (0.15 \text{ m})^2 (2.0 \text{ A})}{2((0.30 \text{ m})^2 + (0.15 \text{ m})^2)^{3/2}} = 7.49 \times 10^{-7}$ T. By the RHR it points out of the page.

The large loop creates a field $\frac{\mu_0 (0.25 \text{ m})^2 (2.0 \text{ A})}{2((0.30 \text{ m})^2 + (0.25 \text{ m})^2)^{3/2}} = 1.319 \times 10^{-6}$ T. By the RHR it points into the page.

The net magnetic field at the center is 5.70×10^{-7} T pointing into the page.