Current and Resistance

SP212 - General Physics II

United States Naval Academy
Annapolis

Current is the flow of charge. The current passing through a surface is the rate at which charge is passing through the surface. Current is measured in units of amperes, where 1 A=1 C/s.

$$I = \left| \frac{dQ}{dt} \right|$$ Current is the flow of charge

- I is the current passing through a surface [units are amperes, A]
- $\frac{dQ}{dt}$ is the rate at which charge is passing through this surface [units are coulombs per second, C/s]

While current is a scalar, we usually associate a direction with it. The direction of the (conventional) current is the direction of the flow of positive charge.

- We know that (negative) electrons are what actually move in a current-carrying wire.
- Suppose current is moving to the right: Actually, electrons are moving to the left.
- Blame Ben Franklin, not me!

$I = qnAv_\mathrm{d}$

- I is the current passing through a wire. [units are amperes, A]
- q is the (absolute value of the) charge of the moving charge carrier. (For a typical metal electrons are the carrier so $q = e = 1.60 \times 10^{-19}$ C.) [units are coulombs, C]
- n is the electron number density of the metal. (The number of mobile electrons per cubic meter.) [units are per cubic meter, $1/\mathrm{m}^3$]
- A is the cross-sectional area of the wire [units are square meters, m^2]
- v_d is the drift speed of electrons in the wire (the magnitude of their average velocity). [units are meters per second, m/s]
  - The drift speed is much smaller than the average speed of the electrons. Electrons move quite fast, but frequently collide with nuclei and other electrons and change direction. There is a much slower drift in one particular direction. That is the drift speed.

J represents the current density, or current per area.

$$I = \int_S \vec{J} \cdot \hat{n} \, dA \qquad \text{Current}$$

$$I = JA \qquad \text{Uniform } J \text{ normal to surface}$$

- J is the current density. [units are amperes per square meter, $\mathrm{A/m^2}$]
- A is the area of the surface through which the current passes (typically, this is the cross-sectional area of a wire). [units are square meters, $\mathrm{m^2}$]
- I is the current passing through the surface. [units are amperes, A]
- $\hat{n}$ is a unit normal for the surface.

When we have a uniform current density passing normally through a flate surface, this is just $J = I/A$.

Suppose that a voltage is applied across an object; this causes a current to flow through the object. We can define the object's resistance as $R = V/I$. The unit of resistance is the ohm, symbolized Ω. $1\ \Omega = 1\ \text{V/A}$.

An object is said to obey Ohm's law (or can be said to be ohmic) if its resistance $R = V/I$ is a constant that does not depend upon the size or direction of the potential difference.

Some objects (such as diodes) do NOT follow Ohm's law. But we will focus on objects, called resistors, that do obey Ohm's law. These objects will have a fixed resistance (determined by the object itself), and we can write $V = IR$: the voltage across the resistor is equal to the current flowing through it times its resistance.

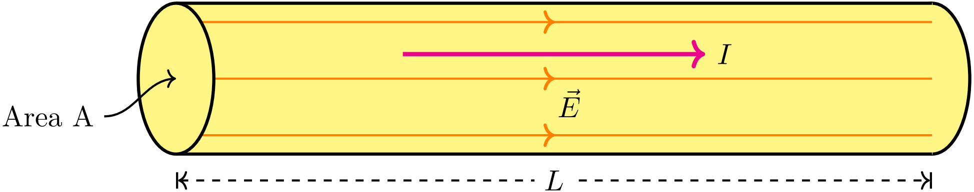
Current (positive charge) will move from higher V toward lower V, so the voltage across a resistor in the direction of the current is $\Delta V = -IR$. (This is the potential at the current output side minus the potential at the current input side.)

There must be an electric field inside a wire that causes the current to flow. (We have previously learned that the electric field inside a conductor at equilibrium is always zero. But if current is flowing through the wire then it is NOT at equilibrium and it needs an electric field.) There will be a ratio relating the current density to the electric field that is known as the resistivity. This resistivity is a property of the material that the wire is made of.

$\vec{E} = \rho \vec{J}$ — Microscopic version of Ohm's Law

- $\vec{E}$ is the electric field inside the wire. [units are newtons per coulomb or volts per meter, N/C or V/m]
- ρ is the resistivity of the material the wire is made of. [units are ohm meters, $\Omega \cdot$m]
- $\vec{J}$ is the current density in the wire. [units are amperes per square meter, A/m^2]

We have a straight wire of length L and cross-sectional area A. It is made of a material whose resistivity is ρ.



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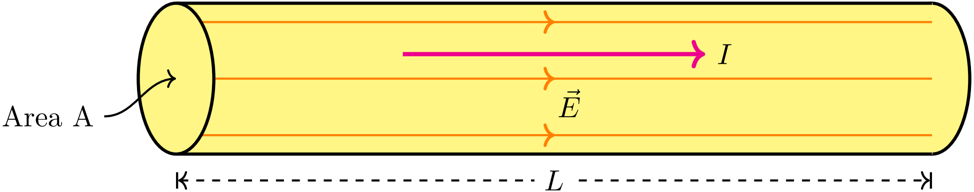

The resistance of this wire is $R = V/I$. We can say that $V = EL$: the voltage across the wire is the electric field inside it time its length. This is just from $\Delta V = -\int \vec{E} \cdot d\vec{l}$.

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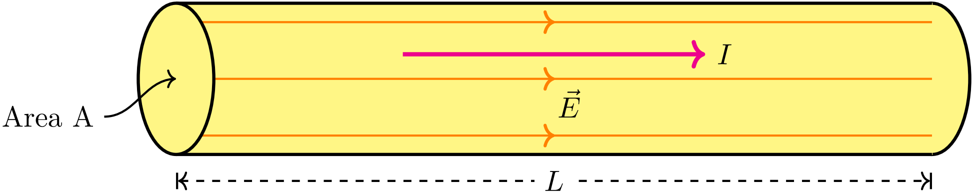

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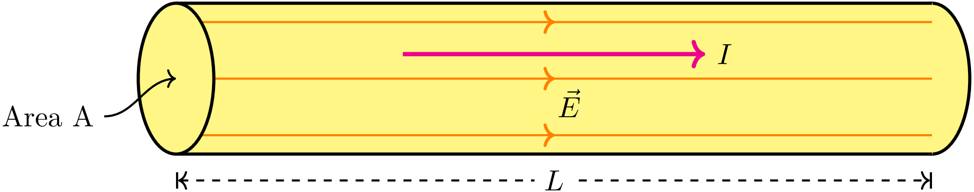

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$$R = \frac{V}{I} = \frac{EL}{(EA/\rho)} = \frac{\rho L}{A}$$

$R = \frac{\rho L}{A}$ Resistance

- R is the resistance of a resistor. [units are ohms, Ω]
- ρ is the resistivity of the material that the resistor is made of. [units are ohm meters, $\Omega \cdot$m]
- L is the length of the resistor (along the direction along which the current will flow) [units are meters, m]
- A is the resistor's cross-sectional area. [units are square meters, m^2]

We see that resistivity is a property of a material. Resistance is a property of an object (called a resistor) made out of that material.

Example:

Consider a list of equations introduced in this lesson: A, E, I, J, L, n, R, v_d, ρ.

(a) Which of these are properties of a particular material? (This means that the variable will be the same for all of a given material (at least at the same temperature), no matter what shape a particular object made of that material is.)

(b) Which of these are properties of a particular object? (This means that the variable will be constant for a given object no matter what voltage is applied across the object.)

(c) Which of these variables will scale with the potential difference (voltage) applied across the object?

Solution:

(a) Which of these are properties of a particular material? (This means that the variable will be the same for all of a given material (at least at the same temperature), no matter what shape a particular object made of that material is.)

ρ and n

(b) Which of these are properties of a particular object? (This means that the variable will be constant for a given object no matter what voltage is applied across the object.)

L, A, and R

(c) Which of these variables will scale with the potential difference (voltage) applied across the object?

v_{d}, J, and I