

Theory

A. Reporting uncertainty

After performing a measurement we might report a length as $x = (0.642 \pm 0.016)$ m. In this case 0.642 m is our best estimate of the length while 0.016 m is the uncertainty or error bar on the measurement. Another way this is sometimes written is as $x = 0.642(16)$ m: the error bar is put at the end in parentheses and the last digit in the error bar lines up with the last digit in the given value.

- My suggestion: when the first digit of your uncertainty is a 1, keep two significant figures in the uncertainty. If the first digit is a 2 or higher you can round it off to only a single significant figure.
- In the measured value, always keep enough significant figures so that the decimal positions of the last sig. fig. in the value and uncertainty match.

Examples using the above guidelines would include (189.2 ± 1.3) m (7.9 ± 0.5) N, (1.234 ± 0.008) s, etc.

B. Error propagation

We may need to perform calculations using values with uncertainty in them. Let us start with $f(x, y, z)$: f is a function of measured variables x , y , and z . Each of these measured variables has an uncertainty Δx , Δy , or Δz . What is Δf ? It turns out that

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x} \Delta x\right)^2 + \left(\frac{\partial f}{\partial y} \Delta y\right)^2 + \left(\frac{\partial f}{\partial z} \Delta z\right)^2}$$

The above formula is always valid, but there are a few common cases where the result is worth knowing. Let us treat $A \pm \Delta A$ and $B \pm \Delta B$ as measured values with uncertainties and n as a constant number with no uncertainty.

Addition or subtraction, $C = A \pm B$:

$$\Delta C = \sqrt{(\Delta A)^2 + (\Delta B)^2}$$

Multiplication or division, $C = A \cdot B$ or $C = A/B$:

$$\frac{\Delta C}{C} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$$

Multiplication by a constant: $C = nA$:

$$\Delta C = n\Delta A$$

Constant exponent: $C = A^n$:

$$\frac{\Delta C}{C} = n \frac{\Delta A}{A}$$

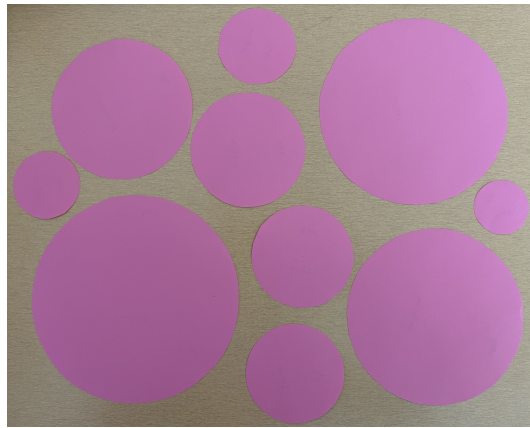
Suppose we have a formula: $m = \sigma\pi\frac{d^2}{4}$. First, rearrange this formula to solve for π . Then derive a formula that give $\frac{\Delta\pi}{\pi}$ as a function of $\frac{\Delta d}{d}$, $\frac{\Delta m}{m}$, and $\frac{\Delta\sigma}{\sigma}$ and enter this in the lab report..

Lab

C. Error propagation

We will have a ream of colored paper. Count out 5 sheets of paper. Using our laboratory balance, measure the mass 5 sheets of paper. This balance says that the uncertainty is ± 0.05 g. Divide by 5 to determine the mass of a single sheet of paper. Now calculate the area mass density of a sheet of paper; this is simply the mass of the paper divided by its area (i.e. $\sigma = m/A$). A sheet of paper is 8.5 inches by 11.0 inches, and 1 inch = 0.0254 m (exactly). You can treat the area of the sheet of paper as exact (no uncertainty). Determine the mass density and its uncertainty and enter them in the lab report.

Using a compass, draw out 10 circles on your sheets of paper. Aim for circles with diameters ranging from about 4.0 cm up to 20 cm, with a range of roughly evenly spaced diameters. You can save a couple of sheets of paper by drawing multiple small circles on the same sheet, but none of the circles should overlap. Now carefully cut each circle out of the paper - try to cut as smoothly along the drawing as possible to produce a nice circle.



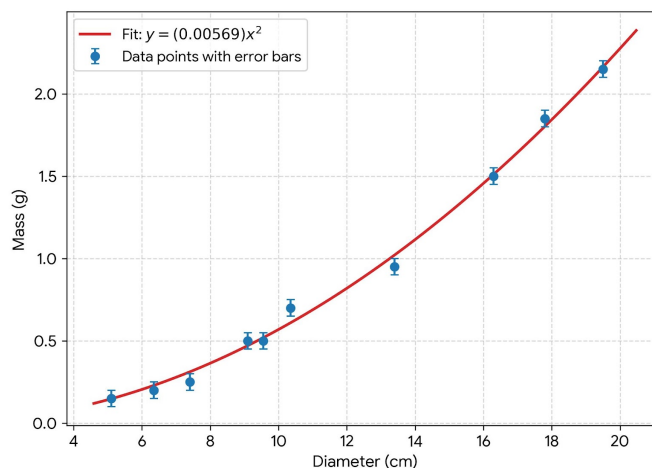
Measure the mass of each of your 10 circles and record the values. We will assume that the uncertainty for each measurement is 0.05 g.

Measure the diameter of each of the 10 circles. Be sure that you are measuring across a full diameter. Hint: the compass needle should have left a small hole at the center of the circle, so your ruler should just touch that small hole as you measure a diameter. Record each diameter and its uncertainty. Measure each diameter twice at different lengths across the circle. If these measurements vary, average them and treat one-half of the spread in measurements (the largest diameter measured minus the smallest diameter measured) as the uncertainty. If all of the measurements are equal or if they differ by only 0.05 cm then the uncertainty will be the smallest distance that can be measured; with care you should be able to measure with a precision of 0.05 cm.

Complete the first table in the lab report: for each of your ten circles calculate an experimental value for π as well as an uncertainty $\Delta\pi$. As usual, keep one significant digit in $\Delta\pi$ (unless the first digit is a 1, then keep two significant digits) and keep enough significant digits in π so that the decimal positions of the last digit in the value and uncertainty match.

D. Fitting

Make a plot showing the mass of your circles as a function of their diameter. The y -axis (mass) data points should have error bars on them. There should be a Gemini Gem set to do this as a weighted least squares fit. Fit your data to $y = Ax^2$. Gemini will give the fit value A as well as its standard error.



Determine the formula that gives your calculated value of π in terms of A (the fit parameter) and σ . Then determine the formula that gives $\frac{\Delta\pi}{\pi}$ in terms of $\frac{\Delta A}{A}$ and $\frac{\Delta\sigma}{\sigma}$. (As a note, π in this formula corresponds to your calculated value, which may differ a bit from the real value.) Then determine your calculated value of $\pi \pm \Delta\pi$ and give that with the standard significant digit conventions.

E. Chi-squared

How does a weighted least-squares fitting routine work? Basically the fit value of our parameter is the one that minimizes the sum of the square of the distance between the fit line and each data point divided by the error bar at that point.

In the second table in the lab report, you will repeat your x , y , and y -error data. Then, for each data point determine the value of the best fit curve at the point: this is just $m_{\text{fit}} = Ax^2$ where A is the best fit value and x is the x -axis value (diameter) for that data point. Then, for each data point determine $\left(\frac{m - m_{\text{fit}}}{\Delta m}\right)^2$: the mass (y -axis) value minus its fit value, divided by the error bar and then squared (this is a unitless number). Keep two digits after the decimal point for each entry. Sum these ten values and enter them in the spot below as χ^2 (pronounced chi-squared). This is the quantity that is minimized in the fitting. Finally, the number of degrees of freedom is the number of data points in your graph (10 in your case) minus the number of fitting parameters (1 in your case). Divide χ^2 by 9 (your number of degrees of freedom) and enter this in the bottom row of the table. This is referred to as the reduced chi-squared for the fit (symbolized $\tilde{\chi}^2$).

Reduced chi-squared is the measure of the goodness of fit that is appropriate for a physicist to use. You may have seen R^2 in an excel fit and heard it used as a measure of the goodness of fit in other contexts. **No self-respecting physicist can ever cite R^2 to describe their fits. That statistic is meant for social scientists. It is not meant for us.** A good fit will have a $\tilde{\chi}^2$ that is close to 1. A $\tilde{\chi}^2$ that is much smaller than 1 almost always means that you overestimated your error bars or that you are using an inappropriately large number of fitting parameters. A $\tilde{\chi}^2$ much larger than 1 usually indicates a very poor fit.

Your fit parameter A came with a standard error ΔA . We are going to explore that now. The key is that changing the fit parameter for its best fit value by one standard error increases the fit χ^2 by 1. $m_{\text{fit} \pm \Delta} = (A \pm \Delta A)x^2$ is what the fit value would be if the fit parameter were changed by one ΔA . Go back through each data point and find $\left(\frac{m - m_{\text{fit} \pm \Delta}}{\Delta m}\right)^2$ and then sum each column. You should find that χ^2 for each of these cases is about 1 bigger than in the case with the best fit.

Error propagation

Derive a formula that give $\frac{\Delta\pi}{\pi}$ as a function of $\frac{\Delta d}{d}$, $\frac{\Delta m}{m}$, and $\frac{\Delta\sigma}{\sigma}$.

Give σ and $\Delta\sigma$ here:

m (g)	Δm (g)	d (cm)	Δd (cm)	π	$\Delta\pi$

Fitting

Formula that gives your calculated value of π in terms of A and σ :

Formula that gives $\frac{\Delta\pi}{\pi}$ in terms of $\frac{\Delta A}{A}$ and $\frac{\Delta\sigma}{\sigma}$:

Calculated value of $\pi \pm \Delta\pi$:

Chi-squared

d (cm)	m (g)	Δm (g)	$\left(\frac{m - m_{\text{fit}}}{\Delta m}\right)^2$	$\left(\frac{m - m_{\text{fit}+\Delta}}{\Delta m}\right)^2$	$\left(\frac{m - m_{\text{fit}-\Delta}}{\Delta m}\right)^2$
χ^2 (Sum of the above)					
$\tilde{\chi}^2$ (Reduced chi-squared)					