Question #1:

(a) $|0\ 0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ is the only state with $s = 0$. So

$$\begin{aligned}\langle 0\ 0|\Psi\rangle &= \frac{1}{\sqrt{2}}(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|)\frac{1}{\sqrt{3}}\left[|\uparrow\downarrow\rangle - (1+i)|\downarrow\uparrow\rangle\right] \\ &= \frac{1}{\sqrt{6}}\left[1 + (1+i)\right] \\ &= \frac{2+i}{\sqrt{6}}\end{aligned}$$

$$\begin{aligned}P_{s=0} &= \left|\frac{2+i}{\sqrt{6}}\right|^2 \\ &= \frac{4+1}{6} = \frac{5}{6}\end{aligned}$$

(b)

$$\begin{aligned}\langle 0\ 0|\Psi\rangle &= \frac{1}{\sqrt{2}}(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|)\frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle e^{-i\omega_0 t} - |\downarrow\uparrow\rangle e^{+i\omega_0 t}] \\ &= \frac{1}{2}\left(e^{-i\omega_0 t} + e^{+i\omega_0 t}\right) = \cos(\omega_0 t)\end{aligned}$$

$$\begin{aligned}\langle 1\ 0|\Psi\rangle &= \frac{1}{\sqrt{2}}(\langle\uparrow\downarrow| + \langle\downarrow\uparrow|)\frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle e^{-i\omega_0 t} - |\downarrow\uparrow\rangle e^{+i\omega_0 t}] \\ &= \frac{1}{2}\left(e^{-i\omega_0 t} - e^{+i\omega_0 t}\right) = -i\sin(\omega_0 t)\end{aligned}$$

$\boxed{P_{s=0} = |\cos(\omega_0 t)|^2 = \cos^2(\omega_0 t) \text{ chance of measuring } 0}$

$\boxed{P_{s=1} = |-i\sin(\omega_0 t)|^2 = \sin^2(\omega_0 t) \text{ chance of measuring } 2\hbar^2}$

$\langle S^2\rangle = \cos^2(\omega_0 t)\cdot 0 + \sin^2(\omega_0 t)\cdot 2\hbar^2$

$\boxed{\langle S^2\rangle = 2\hbar^2\sin^2(\omega_0 t)}$

Question #2:

(a)

$$|\Psi\rangle = |s = 3/2 \quad m = +1/2\rangle$$
$$= \sqrt{\frac{2}{5}}|s_1 = 3/2 \quad m_1 = 3/2\rangle|s_2 = 1 \quad m_2 = -1\rangle$$
$$+ \sqrt{\frac{1}{15}}|s_1 = 3/2 \quad m_1 = 1/2\rangle|s_2 = 1 \quad m_2 = 0\rangle$$
$$- \sqrt{\frac{8}{15}}|s_1 = 3/2 \quad m_1 = -1/2\rangle|s_2 = 1 \quad m_2 = 1\rangle$$

$$\boxed{\frac{2}{5} \text{ chance of } +\frac{3\hbar}{2}, \frac{1}{15} \text{ chance of } +\frac{\hbar}{2}, \text{ and } \frac{8}{15} \text{ chance of } -\frac{\hbar}{2}}$$

(b)

$$|\Psi\rangle = |s_1 = 3/2 \quad m_1 = -1/2\rangle|s_2 = 1 \quad m_2 = 1\rangle$$
$$= \sqrt{\frac{3}{10}}|s = 5/2 \quad m = 1/2\rangle - \sqrt{\frac{8}{15}}|s = 3/2 \quad m = 1/2\rangle + \sqrt{\frac{1}{6}}|s = 1/2 \quad m = 1/2\rangle$$

$$\boxed{\frac{3}{10} \text{ chance of } \frac{35\hbar^2}{4}, \frac{8}{15} \text{ chance of } \frac{15\hbar^2}{4}, \text{ and } \frac{1}{6} \text{ chance of } \frac{3\hbar^2}{4}}$$

Question #3: $\hat{S}^2 = \left(\hat{S}^{(1)}\right)^2 + \left(\hat{S}^{(2)}\right)^2 + 2\hat{S}_x^{(1)}\hat{S}_x^{(2)} + 2\hat{S}_y^{(1)}\hat{S}_y^{(2)} + 2\hat{S}_z^{(1)}\hat{S}_z^{(2)}$

$\left(\hat{S}^{(1)}\right)^2 |\uparrow\uparrow\rangle = \frac{3\hbar^2}{4}|\uparrow\uparrow\rangle$ and $\left(\hat{S}^{(2)}\right)^2 |\uparrow\uparrow\rangle = \frac{3\hbar^2}{4}|\uparrow\uparrow\rangle$

$\hat{S}_x|\uparrow\rangle = \frac{\hbar}{2}|\downarrow\rangle$ for a single spin. Therefore $2\hat{S}_x^{(1)}\hat{S}_x^{(2)}|\uparrow\uparrow\rangle = 2\left(\frac{\hbar}{2}\right)\left(\frac{\hbar}{2}\right)|\downarrow\downarrow\rangle$

$\hat{S}_y|\uparrow\rangle = \frac{i\hbar}{2}|\downarrow\rangle$ for a single spin. Therefore $2\hat{S}_y^{(1)}\hat{S}_y^{(2)}|\uparrow\uparrow\rangle = 2\left(\frac{i\hbar}{2}\right)\left(\frac{i\hbar}{2}\right)|\downarrow\downarrow\rangle$

$2\hat{S}_z^{(1)}\hat{S}_z^{(2)}|\uparrow\uparrow\rangle = 2\left(\frac{\hbar}{2}\right)\left(\frac{\hbar}{2}\right)|\uparrow\uparrow\rangle$

Putting these together, $\hat{S}^2|\uparrow\uparrow\rangle = \frac{3\hbar^2}{4}|\uparrow\uparrow\rangle + \frac{3\hbar^2}{4}|\uparrow\uparrow\rangle + \frac{\hbar^2}{2}|\downarrow\downarrow\rangle - \frac{\hbar^2}{2}|\downarrow\downarrow\rangle + \frac{\hbar^2}{2}|\uparrow\uparrow\rangle$ which adds up to $\boxed{\hat{S}^2|\uparrow\uparrow\rangle = 2\hbar^2|\uparrow\uparrow\rangle}$. Therefore $|\uparrow\uparrow\rangle$ is an eigenstate of $\hat{S}^2$ with an eigenvalue of $2\hbar^2$. This corresponds to $\hbar^2 s(s+1)$ for $s = 1$.