Question #1: The spinor state of a spin-1/2 particle is given by is given by:

$$\chi(t) = \begin{pmatrix} 0.6e^{+i\omega t} \\ 0.8e^{-i\omega t} \end{pmatrix}$$

(a) Determine, as a function of time, the probability that a measurement of $\hat{S}_z$ will give an answer of $-\dfrac{\hbar}{2}$ and the probability that it will give an answer of $+\dfrac{\hbar}{2}$.

(b) Determine, as a function of time, the probability that a measurement of $\hat{S}_x$ will give an answer of $-\dfrac{\hbar}{2}$ and the probability that it will give an answer of $+\dfrac{\hbar}{2}$.

Question #2: An electron is placed in an oscillating magnetic field: $\vec{B}(t) = B_0\cos(\omega t)\hat{k}$. The Hamiltonian of the system is then

$$\hat{H} = -\frac{\gamma\hbar}{2}B_0\cos(\omega t)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The spinor state of our electron is given by is given by:

$$\chi(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\frac{\gamma B_0}{2\omega}\sin(\omega t)} \\ e^{-i\frac{\gamma B_0}{2\omega}\sin(\omega t)} \end{pmatrix}$$

(a) Confirm that this spinor state obeys the Schrödinger equation (i.e. show that $\hat{H}\chi = i\hbar\frac{d\chi}{dt}$).

(b) Determine $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$ (each may depend upon time).

(c) Determine, as a function of time, the probability that a measurement of $\hat{S}_x$ will give an answer of $-\frac{\hbar}{2}$.

(d) You might notice that at $t = 0$ a measurement of $\hat{S}_x$ will give an answer of $+\frac{\hbar}{2}$ with 100% probability (i.e. $\chi(t = 0) = \chi_+^{(x)}$). What is the minimum field ($B_0$) needed to force a complete spin flip (so that your answer in part c will reach 100% at some time)?