Question #1: Two non-interacting particles with the same mass are placed in a 1D simple harmonic oscillator potential. The stationary states could be written as

$$\psi_{n_1}(x_1)\psi_{n_2}(x_2) \quad \text{with energy} \quad E_{n_1,n_2} = \hbar\omega(n_1+n_2+1)$$

where n_1 and n_2 are non-negative integers and $\psi_{n_1}(x_1)$ represents a typical single particle stationary state.

In this problem you will be writing out lists of possible states. In all cases, write them as spatial states followed by total spin states. For total spin states you can leave the m quantum number as just an m. You can drop the (x_1) and (x_2) in $\psi_{n_1}(x_1)\psi_{n_2}(x_2)$: if you just write $\psi_0\psi_2$ I will assume that particle #1 is in a $n=0$ state while particle #2 is in a $n=2$ state. Here is a list of possible states that might show up using this nomenclature:

i) $\psi_0\psi_2|0\ 0\rangle$: ($n_1=0$, $n_2=2$; total spin $s=0$ singlet)

ii) $\frac{1}{\sqrt{2}}(\psi_1\psi_2+\psi_2\psi_1)|2\ m\rangle$: (spatially symmetric combo of $n_1=1$, $n_2=2$ and $n_1=2$, $n_2=1$; total spin $s=2$ quintuplet)

iii) $\frac{1}{\sqrt{2}}(\psi_0\psi_3-\psi_3\psi_0)|1\ m\rangle$: (spatially antisymmetric combo of $n_1=0$, $n_2=3$ and $n_1=3$, $n_2=0$; total spin $s=1$ triplet)

a) You have two spin-1/2 particles. List the allowed states and degeneracies for energies between $\hbar\omega$ and $4\hbar\omega$ for the case of two distinguishable particles and also for indistinguishable fermions.

b) You have two spin-1 particles. List the allowed states and degeneracies for energies between $\hbar\omega$ and $4\hbar\omega$ for the case of two distinguishable particles and also for indistinguishable bosons. (Note: Look at the Clebsch-Gordan table for 1×1 and it should be clear that spin states with total spin $s=2$ or $s=0$ are symmetric under exchange while spin states with total spin $s=1$ are antisymmetric under exchange.)

Question #2: In natural units, where $m = \hbar = \omega = 1$, the ground state and first excited state for the 1D simple harmonic oscillator are given by

$$\psi_0(x) = \left(\frac{1}{\pi}\right)^{1/4} e^{-x^2/2} \qquad \text{and} \qquad \psi_1(x) = \left(\frac{1}{\pi}\right)^{1/4} \sqrt{2}\, x\, e^{-x^2/2}$$

You have two spinless, noninteracting particles of the same mass in the same 1D SHO potential. One particle is in the ground state while the other is in the first excited state.

(a) Write out $\psi(x_1, x_2)$ and $|\psi(x_1, x_2)|^2$ if the particles are:
(i) Distinguishable (assuming particle #1 is in the ground state)
(ii) Identical spinless bosons
(iii) Identical "spinless" fermions

(b) $|\psi(x_1, x_2)|^2$ can instead be written as $|\psi(R, r)|^2$, where $R \equiv (x_1 + x_2)/2$ is the center of mass coordinate and $r \equiv (x_1 - x_2)$ is the particle separation. These are
(i) $|\psi(R, r)|^2 = \frac{2}{\pi}\left(R - \frac{r}{2}\right)^2 e^{-(\frac{1}{2}r^2 + 2R^2)}$ for distinguishable particles

(ii) $|\psi(R, r)|^2 = \frac{1}{\pi}\left(4R^2\right) e^{-(\frac{1}{2}r^2 + 2R^2)}$ for identical bosons

(iii) $|\psi(R, r)|^2 = \frac{1}{\pi} r^2 e^{-(\frac{1}{2}r^2 + 2R^2)}$ for identical fermions

For each of the three cases integrate over R to determine the probability density of finding the particles a distance r apart:

$$P(|r|) = 2\int_{-\infty}^{\infty} |\psi(R, r)|^2\, dR$$

where the factor of 2 is because r could be positive or negative. Plot $P(|r|)$ for each of the three cases on the same graph in a new Desmos graph (labeling which is which). Start your graphs at $|r| = 0$ on the x-axis.

You may use these integrals or Wolfram Alpha or similar:

$$\int_{-\infty}^{\infty} e^{-ax^2}\, dx = \sqrt{\frac{\pi}{a}} \qquad \text{and} \qquad \int_{-\infty}^{\infty} x^2\, e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

where $\text{Re}(a) > 0$.

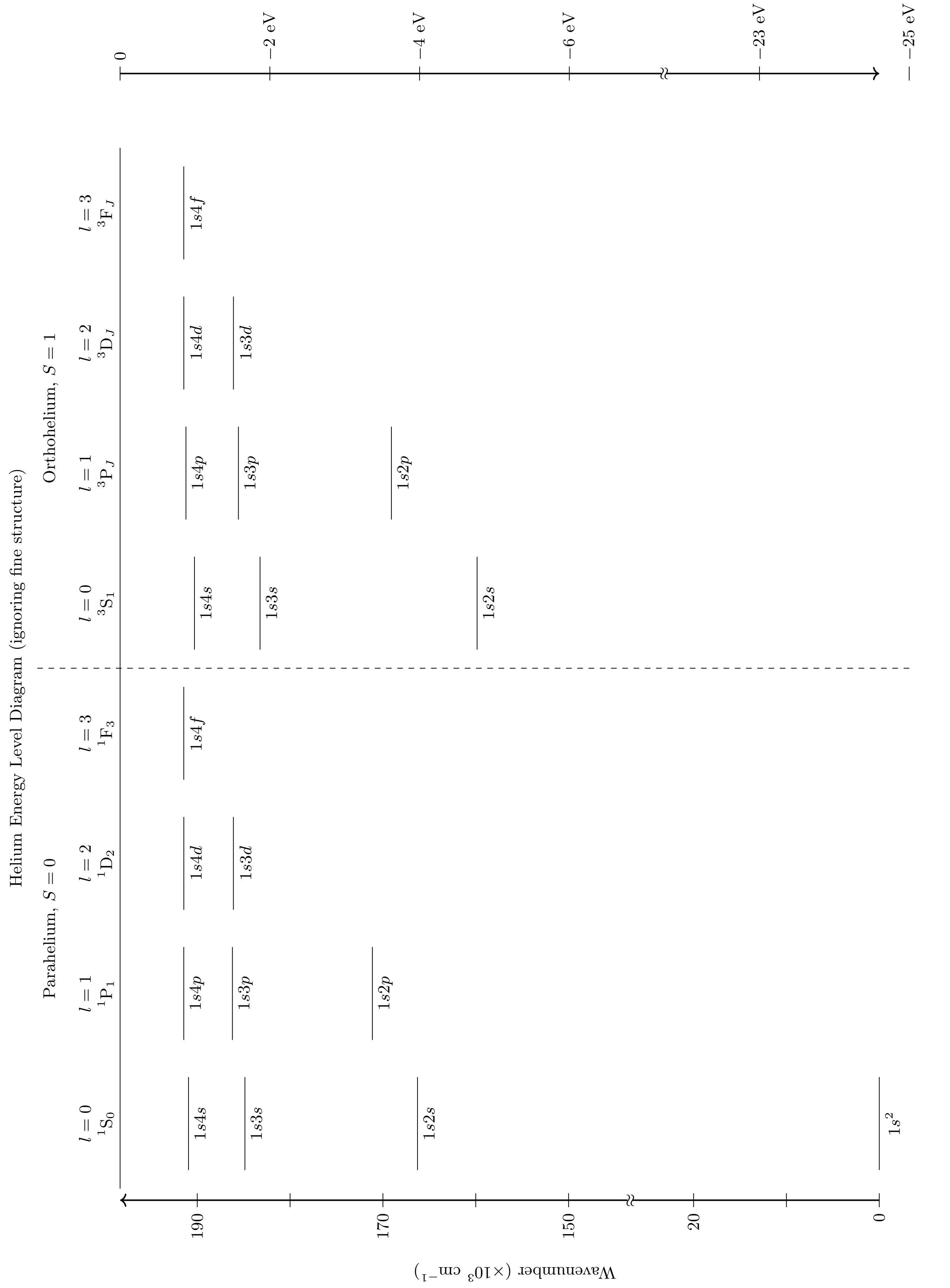
Question 3: (a) Go to the website

`https://physics.nist.gov/PhysRefData/ASD/levels_form.html`

type `He I` in the box next to Spectrum: (this means that we want the spectrum for neutral helium) and then press the Retrieve Data button. This will give the energy levels for neutral helium. Give me the energy level of the 11 levels (6 parahelium and 5 orthohelium) where the second electron is in a $n = 1$, $n = 2$, or $n = 3$ state. Give these levels in units of cm^{-1}, accurate to 5 significant figures, and with the convention that the ground state is set to 0. (Note: orthohelium states have $S = 1$ and therefore a 3 in the upper left of the term symbol; parahelium states have $S = 0$ and therefore a 1 in the upper left of the term symbol. For the orthohelium levels with $l \geq 1$ there will be three levels with different values of J. They will be slightly split in energy due to fine structure. But we are ignoring that for now, and each of the three should be the same when rounded off to 5 significant figures.)

(b) For the $1s2s$ state for each of parahelium and orthohelium, convert these energy levels to units of eV, with double ionization set to 0. (With the ground state at 0, the double ionization energy is 637220 cm^{-1}.) We could roughly model these energies as $(-2^2 \cdot 13.606 \text{ eV})\left(1 + \frac{1}{2^2}\right) + \frac{1}{4\pi\epsilon_0}\frac{e^2}{\langle|\vec{r}_1 - \vec{r}_2|\rangle}$, i.e. two hydrogen-like energies plus an electron-electron repulsion. (The 2^2 in the first term is because there are two protons in the nucleus. The $(1 + 1/2^2)$ is because one electron is in a $n = 1$ state while the other is in a $n = 2$ state. Use this to make a rough estimate for $\langle|\vec{r}_1 - \vec{r}_2|\rangle$ (the average distance between the two electrons) for both the the parahelium and orthohelium states (in units of either nm or Å and to three significant figures). You should start by converting $\frac{e^2}{4\pi\epsilon_0}$ into units of either eV·nm or eV·Å.

(c) In the energy level diagram given on the next page, draw arrows for the 10 allowed transitions that are $n = 3$ to $n = 1$, $n = 2$ to $n = 1$, $n = 2$ to $n = 2$, or $n = 3$ to $n = 2$ (remember: only transitions with $\Delta L = \pm 1$ and $\Delta S = 0$ are allowed). Determine the wavelengths of each of these 10 transitions, in units of nm and to at least four significant figures. Be sure to be clear which wavelengths are from parahelium and which are from orthohelium.


Helium Energy Level Diagram (ignoring fine structure)
Parahelium, $S = 0$
Orthohelium, $S = 1$
$l = 0$ $^1\mathrm{S}_0$
$l = 1$ $^1\mathrm{P}_1$
$l = 2$ $^1\mathrm{D}_2$
$l = 3$ $^1\mathrm{F}_3$
$l = 0$ $^3\mathrm{S}_1$
$l = 1$ $^3\mathrm{P}_J$
$l = 2$ $^3\mathrm{D}_J$
$l = 3$ $^3\mathrm{F}_J$
$1s4s$
$1s3s$
$1s2s$
$1s^2$
$1s4p$
$1s3p$
$1s2p$
$1s4d$
$1s3d$
$1s4f$
0
-2 eV
-4 eV
-6 eV
-23 eV
-25 eV
190
170
150
20
0
Wavenumber ($\times 10^3$ cm^{-1})