

Question #1: Determine the lowest order relativistic correction ($E_r^1 = \frac{-1}{8m^3c^2} \langle \psi_1 | \hat{p}^4 | \psi_1 \rangle$) for the ground state of an infinite square well potential. Express your answer in terms of E_1^0 and mc^2 .

Question #2: Determine the allowed eigenvalues of the $\vec{L} \cdot \vec{S}$ operator when: (a) $l = 0$ and $s = 1/2$, (b) $l = 1$ and $s = 1/2$, and (c) $l = 2$ and $s = 1/2$.

Question #3: This problem will require calculating wavelengths to very high precision, so below I give some well-known constants accurate to seven significant figures. Use these and give your answers also to seven significant figures.

$$hc = 1239.842 \text{ eV} \cdot \text{nm}$$

$$\alpha = 1/137.0360 = 0.007297353$$

$$1 \text{ Ry} \cdot \frac{\mu_e}{m_e} = 13.59828 \text{ eV}$$

(In this last line, I am multiplying 1 Rydberg by the ratio of the reduced mass of an electron-proton system to the bare electron mass. We are calculating so accurately that it matters that the proton is not infinitely massive, so you should use this number instead of 13.6 eV when calculating energies.)

(a) Using Equation 7.69 in your textbook, determine (in units of eV and accurate to seven significant figures) the energies of the three $n = 3$ states of hydrogen including fine structure.

(b) Using Equation 7.69 in your textbook, determine (in units of eV and accurate to seven significant figures) the energies of the two $n = 2$ states of hydrogen including fine structure.

(c) Selection rules require that $\Delta j = 0, \pm 1$. This means that there is no allowed ($n = 3, j = \frac{5}{2}$) to ($n = 2, j = \frac{1}{2}$) transition. This leaves five allowed transitions between $n = 3$ and $n = 2$. Determine (in units of nm and accurate to seven significant figures) the wavelengths of each of these five transitions.