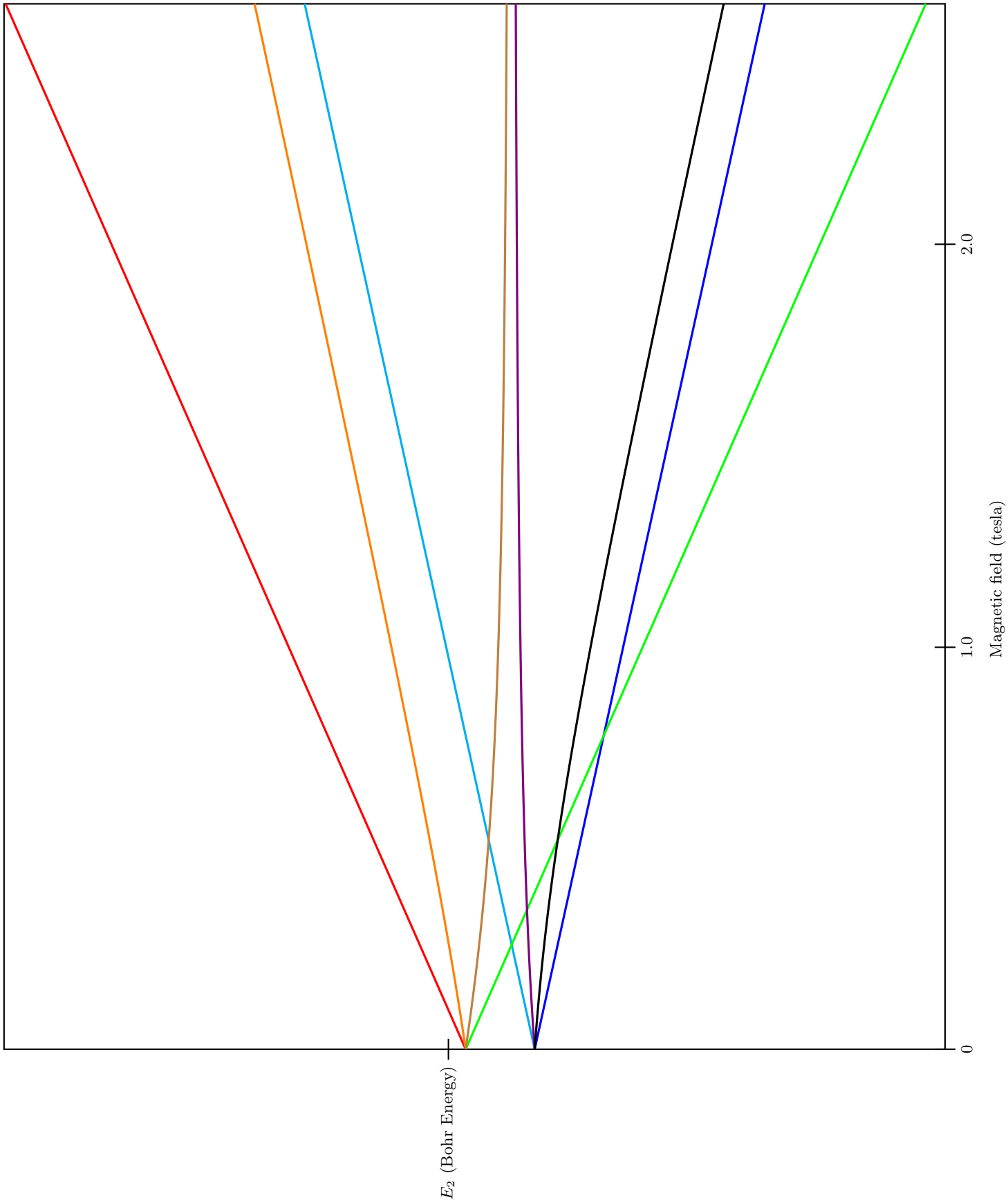
Question #1: (a) There are eight distinct states for hydrogen with $n = 2$. List out these eight states in BOTH the $|n\, l\, m_l\, m_s\rangle$ basis and the $|n\, l\, j\, m_j\rangle$ basis.

n	l	m_l	m_s
2			
2			
2			
2			
2			
2			
2			
2			

n	l	j	m_j
2			
2			
2			
2			
2			
2			
2			
2			

(b) For the $|n\, l\, j\, m_j\rangle$ basis, there will be three distinct pairs of l and j. Determine the Landé g-factor for each of these three cases.

(c) The opposite page shows these eight energies stretching from the weak-field regime, through the intermediate-field regime, into the high-field regime. Label each of these eight energy levels with the appropriate quantum numbers in both the weak-field ($|n\, l\, j\, m_j\rangle$ basis) and high-field ($|n\, l\, m_l\, m_s\rangle$ basis) limits. (In really high fields the $|2\ 1\ -1\ +1/2\rangle$ and $|2\ 1\ +1\ -1/2\rangle$ states merge to first order perturbation theory. You can just identify a pair of two lines as these two.)


E_2 (Bohr Energy)
0
1.0
2.0
Magnetic field (tesla)

Question 2: Suppose you have an electron ($s = 1/2$) in a $l = 1$ orbital. Further suppose that this electron is in a $j = 3/2$, $m_j = +1/2$ state.

(a) Determine the Landé g-factor for this electron using the equation given in your text.

(b) Use the Clebsch-Gordan tables in your book to express this state $|\Psi\rangle = |j = 3/2 \quad m_j = +1/2\rangle$ as a linear combination of the states $|l = 1 \quad m_l = +1\rangle|s = 1/2 \quad m_s = -1/2\rangle$ and $|l = 1 \quad m_l = 0\rangle|s = 1/2 \quad m_s = +1/2\rangle$.

(c) Use your answer from part b to find the expectation values $\langle m_l\rangle \equiv \dfrac{\langle L_z\rangle}{\hbar}$ and $\langle m_s\rangle \equiv \dfrac{\langle S_z\rangle}{\hbar}$ for this electron.[1]

(d) One way to think about the meaning of the g-factor is that

$$g_j \cdot m_j = 1 \cdot \langle m_l\rangle + 2 \cdot \langle m_s\rangle.$$

Confirm that this is true with your answers to part a and c.

Question 3: Suppose that we have two spin-1 particles. These two particles interact via the *Heisenberg Hamiltonian*: $\hat{H} = -J\vec{S}_1 \cdot \vec{S}_2$. (Here J is a constant; it has nothing to do with angular momentum.) Determine each of the eigenvalues of this Hamiltonian (there will be three).

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[1] I'll admit that this is unusual notation: we usually take the expectation value of physical observables, not the quantum numbers associated with them. But I think that this notation is instructive here.