Background

A. The Fabry-Perot etalon

This experiment uses a Fabry-Perot etalon in order to measure frequency shifts with great accuracy. A Fabry-Perot etalon is shown in Figure 1. Light enters a cavity consisting of two glass plates separated by distance d that reflect most light but transmit a little at each reflection. Each extra round trip through the cavity adds a path length difference $2d\cos\alpha$ where α is the angle of the light ray. Constructive interference occurs when $2d\cos\alpha = m\lambda$ for any integer m. The mercury lamp is like a point source, so light is diverging when it reaches the etalon. For this reason, there will always be some light that fulfills the interference condition. A lens focusing the light turns the interference fringes into concentric circles. The highest order (highest m) is the smallest circle nearest the center, with other circles represent each successively smaller m. The spacing d is large in comparison to λ so the highest order m will be quite large - in the ballpark of 10000 for this experiment. But we don't actually need to know this number precisely.

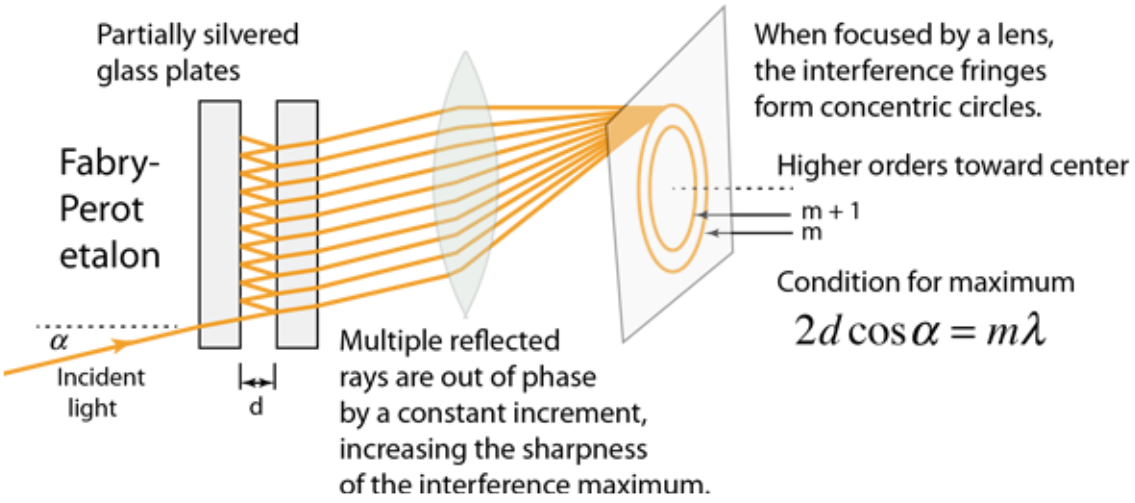

FIG. 1: Diagram of a Fabry-Perot etalon. I took this figure from the HyperPhysics website at http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/fabry.html

So the inner most ring fulfills the condition $2d\cos\alpha_1 = m\lambda$ for some very large m. The second ring is one order lower: it fulfills the condition $2d\cos\alpha_2 = (m-1)\lambda$. Now if we were to make the wavelength of the light smaller (make the frequency bigger) each ring would move outward. It can be shown that if we increase the frequency of the light by an amount $\Delta f = \dfrac{c}{2d}$ the inner most circle will shift to the exact spot where the second circle originally had been (while an additional higher order will appear near the center. The key is that by observing a splitting in these

peaks we can measure frequency shifts to a much higher precision that we know the original frequency. This difference Δf is known as the free spectral range of the etalon. We will use two etalons in this lab: one has a free spectral range of 74.9 GHz while the other has a free spectral range of 59.9 GHz.

B. The Zeeman Effect

The Zeeman effect is the splitting of an atomic spectral line into multiple components due to the application of a magnetic field. Electrons have a magnetic dipole moment, so that an applied field adds an energy $\Delta E = -\vec{\mu} \cdot \vec{B}$. Originally degenerate energy levels will be split, which leads to the splitting of the spectral lines. In the weak-field Zeeman effect the magnetic dipole moment of an electron is related to its m_j quantum number, which describes the projection of the electron's total angular momentum along the field direction.

1. g-factors

Imagine a single electron orbiting a hydrogen nucleus. We will start by considering a classical oversimplification and treat the electron as a point particle of mass m_e and charge $-e$ moving in a classical circular orbit of radius r around the proton at a constant speed v. Let's say that the electron is moving clockwise in the plane of the page and that out of the page is defined as the $+z$ direction. The angular momentum of this particle will be $l_z = -m_e v r$ (directed into the page because the electron is moving clockwise). Because electrons are charged, this motion will also create a magnetic dipole moment. The electron will orbit the proton with a period of $T = \dfrac{2\pi r}{v}$. We can think of this as an average current moving in the circle of $I = \dfrac{e}{T} = \dfrac{ev}{2\pi r}$. This current will be counter-clockwise (opposite the electron's motion) because electrons are negatively charged. The magnetic dipole moment will be $\mu_z = I\pi r^2 = \dfrac{evr}{2}$. In our work so far, we have used r and v to represent the electron's radius and speed but these are not fundamental constants. So we will eliminate them and write $\mu_z = -\dfrac{e}{2m_e} l_z$: the magnetic dipole moment is proportional to the angular momentum with proportionality constant $-\dfrac{e}{2m_e}$. Now we will assume that $l_z = m_l \hbar$ where m_l is a dimensionless number. This gives $\mu_z = -\mu_B m_l$ where

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24} \text{ J/T} = 0.05788 \text{ meV/T}$$

is the Bohr magneton.

The previous discussion was an extreme oversimplification treating an electron as purely classical. A full quantum mechanical treatment ends up multiplying this by a ratio known as the g-factor.

For orbital angular momentum:

$$\hat{\mu}_z = -g_l \frac{\mu_B}{\hbar} \hat{L}_z \quad \text{(operator)} \qquad \text{or} \qquad \mu_z = -g_l \mu_B m_l \quad \text{(eigenvalues)}$$

While for spin angular momentum:

$$\hat{\mu}_z = -g_s \frac{\mu_B}{\hbar} \hat{S}_z \quad \text{(operator)} \qquad \text{or} \qquad \mu_z = -g_s \mu_B m_s \quad \text{(eigenvalues)}$$

The g-factor for orbital angular momentum is $g_l = 1$ (the classical oversimplification gives the correct answer) while for spin angular momentum $g_s \approx 2.0023$ (we will round this off to $g_s \approx 2$).

2. Addition of angular momentum

Because an electron in an atom will have both orbital and spin angular momentum, we will have to consider the total angular momentum: $\vec{J} = \vec{L} + \vec{S}$. The quantum number j can take values between $|l - s|$ and $l + s$ in steps of 1:

$$j = |l - s|, |l - s| + 1, \ ...., \ l + s - 1, \ l + s$$

The relationship between angular momentum and magnetic dipole momentum is much as before

$$\hat{\mu}_z = -g_j \frac{\mu_B}{\hbar} \hat{J}_z \quad \text{(operator)} \qquad \text{or} \qquad \mu_z = -g_j \mu_B m_j \quad \text{(eigenvalues)}$$

The g-factor for total angular momentum is known as the Landé g-factor and is given by

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

You'll notice that in this last equation I switched from using lower case letters for my quantum numbers to using upper case letters. I do that because going forward we will be talking about the total angular momentum of an atom with many electrons instead of a single electron and it is thus conventional to use capital letters. You can check that in the limit $L = 0$ we have $g_J = g_S = 2$ and in the limit $S = 0$ we have $g_J = g_L = 1$.

The angular momentum states of atoms are typically described by their term symbol: ${}^{2S+1}L_J$ where $2S + 1$ and J are represented by numbers while L

is represented by a letter. These letters are S ($L = 0$), P ($L = 1$), D ($L = 2$), F ($L = 3$), G ($L = 4$), ... (continuing alphabetically but skipping J, P, and S). Therefore ${}^4F_{3/2}$ is the term symbol for $S = 3/2$, $L = 3$, $J = 3/2$, etc.

3. Selection rules

It turns out that transitions are not allowed between all possible energy levels in an atom. The light emitted in typical atomic spectra comes from electric dipole interactions. It is possible to use the commutation relations between the electric dipole operator and angular momentum operators to show that such transitions are allowed only when certain selection rules are obeyed. This will be covered in class, but for now I will just state without proof the relevant selection rules.
(a) The l-value of one electron changes by ± 1
(b) $\Delta L = 0,\ \pm 1$
(c) $\Delta S = 0$
(d) $\Delta J = 0,\ \pm 1$
(e) $\Delta M_J = 0,\ \pm 1$
(f) $J = 0$ to $J = 0$ transitions are forbidden
(g) $M_J = 0$ to $M_J = 0$ transitions are forbidden when the sum of the J values in the initial and final states is even.

C. An Example: the 404.7 nm violet line in mercury

You are going to measure the Zeeman effect on both the 546.1 nm green line and the 435.8 nm blue line of the mercury spectrum. As an example, I will do some analysis for the 404.7 nm line in mercury. Interestingly, all three of these lines are transitions from a higher energy state with electron configuration [Xe]$4f^{14}5d^{10}6s^1 7s^1$ to a lower energy state with electron configuration [Xe]$4f^{14}5d^{10}6s^1 6p^1$; the different energies are due to different angular momentum values. This 404.7 nm violet line comes from a ${}^3S_1 \rightarrow {}^3P_0$ transition. The upper level is ${}^3S_1$, so $S = 1$, $L = 0$, and $J = 1$. This level will have a g-factor of $g_J = 2$. An applied field will split this into three levels, with $M_J = -1$, 0, and +1. The lower level is ${}^3P_0$, so $S = 1$, $L = 1$, and $J = 0$. This level does not have a definable g-factor because $J = 0$. It will not split in an applied field.

This transition has $\Delta L = +1$, $\Delta S = 0$, and $\Delta J = -1$. These are allowed under selection rules (a)-(c), so this transition is allowed. There would be three possible transitions: all have $\Delta M_J = 0,\ \pm 1$ so all three are allowed by selection rule (d). The sum of the initial and final J is 1, which is odd. Therefore the $M_J = 0$ to

$M_J = 0$ transition is allowed. Therefore selection rules leave us with three possible transitions. I've labelled them as (i) through (iii): we'll determine their energies (where E_0 is the energy difference in zero field).

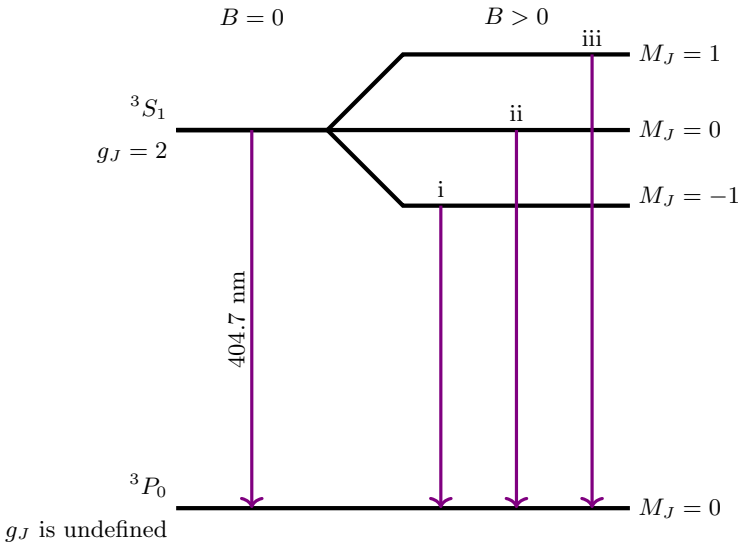

FIG. 2: Energy level diagram for the 404.7 nm violet line in mercury under a magnetic field. Selection rules allow for three possible transitions.

(i): $\Delta E = E_0 + [2(-1) - 0)]\mu_B B = E_0 - 2\mu_B B$ $\quad \Delta M_J = +1$
(ii): $\Delta E = E_0 + [2(0) - 0)]\mu_B B = E_0$ $\quad \Delta M_J = 0$
(iii): $\Delta E = E_0 + [2(+1) - 0]\mu_B B = E_0 + 2\mu_B B$ $\quad \Delta M_J = -1$

Therefore an applied magnetic field will split this line into three lines. If we look at this line passing through a Fabry-Perot etalon, each circle will turn into three circles. This is roughly demonstrated below in Figure 3. You might notice in my schematic that, in addition to each circle splitting into three, a new circle appears in the middle. This is because for the higher energy (shorter wavelength) line that we've created it is now possible to get constructive interference through the Fabry-Perot for an even larger m than our previous maximum. But in practice these new circle coming from the center in a magnetic field tend to be very broad.

Calibrating the magnetic field

Our first step will be to calibrate the magnetic field. The mercury pencil lamp should be carefully removed from the magnet. You should then open the cold water valve at the sink to allow a steady stream of cooling water to pass through the magnet. I'd suggest placing wallets and cell phones on the table on the other side of the room - I don't think that the stray magnetic fields will be large enough to

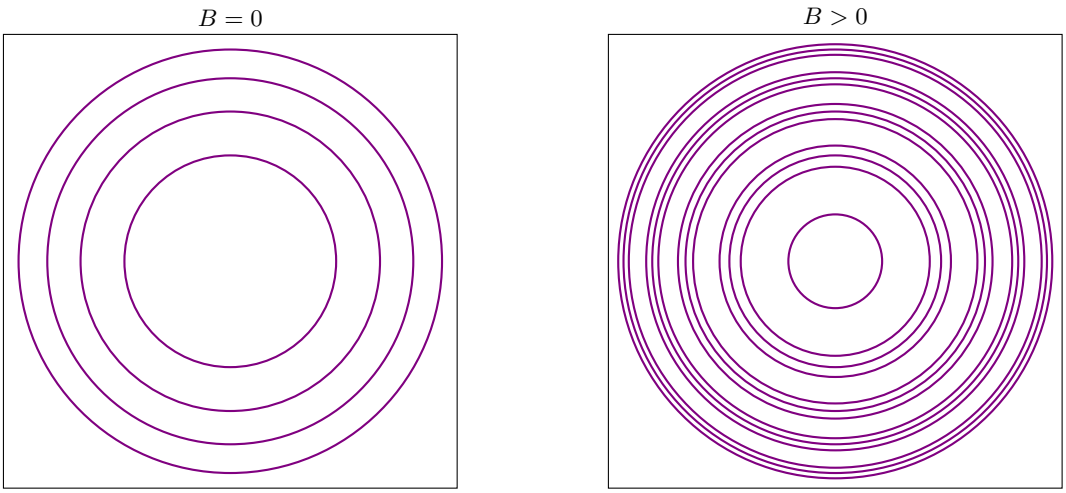

FIG. 3: Schematice showing the splitting of the 404.7 nm violet line in mercury under a magnetic fields as demonstrated by passing the light through a Fabry-Perot etalon.

erase a credit card, but I'm still not going to take that chance. Then turn on the magnet power supply and slowly turn both the current and voltage coarse knobs up until it reaches the maximum value: a current of about 29 A. Turn on the gaussmeter (which measures magnetic field). I suggest taping the end of the probe to something sturdy like a thick cable tie so that it can be better lowered into the magnet bore. Zero it when it is far from the magnet. Then insert the probe tip down into the center of the magnet. Your goal is now to rotate the probe until the field reading is a maximum (we want the probe oriented so that the direction that it measures is aligned with the direction of the field). Looking from the side, keep the probe near the middle of the magnet bore. Slowly rotate around with one eye on the gaussmeter. When you have maximized the reading you should record this value. Then lower the current (by lowering both the current and voltage knobs) and repeat. I'd suggest taking about 20 data points, so steps of about 1.5 A would work well. The magnet has a bit of *hysteresis*: the field at a given current depends upon whether the current has been increasing or decreasing. We'll solve this by taking all data starting at the highest field and then working your way down as you lower the field. When you are finished plot your data using either LoggerPro or the Gemini Gem. Axes should be appropriately labelled with units. Fit your result to a quadratic.

Taking data: the green mercury line

The mercury pencil lamp should be returned to the center of the magnet. We'll start measuring the green line. So the green filter and the (larger) air-gapped Fabry-

Perot etalon should be in the optics line. We will use the ThorLabs camera; it has a USB that needs to be plugged into the computer. Then open the ThorImage CAM program.Remove the lens cap from the camera and the two caps on the Fabry-Perot etalon. Put on goggles to protect your eyes from UV light then turn on the mercury light source. Turn the sink cold water on to provide a steady flow to the magnet. Turn the magnet power supply on. Rotate the polarizer until the 45° mark is aligned with the line at the top (45° allows some light from both polarization states to pass through; we will use it as a stand-in for unpolarized light).

Press the Live button in ThorImage. Maybe then press the Auto Exposure button so that you see clear rings; your instructor can give advice here. You will want several rings clearly visible and centered. Your instructor can help you here. Adjust the Fabry-Perot etalon to center the pattern. Move the lens a little to make the image as sharp as possible. Move the camera zoom to change the size. Adjust the capture time to add statistics if needed. Carefully turn the magnet power supply up: you need to adjust both the voltage and current knobs. The knobs need to move at roughly the same rate: if, for example, you see that increasing the voltage knob does nothing then you need to increase the current knob and vice versa. Slowly turn both up to maximum power. This should be close to 30 A, though if you only hit 28 A or 29 A as your maximum that is fine. There will be bands of rings: ideally you want to be able to make out all nine rings. Move the lens a little if needed to make the nine rings as sharp as possible.

Press Live again to turn Live off. The Capture Configuration settings are at the bottom. Give a good Prefix Name, Capture Mode should be Single Image, and Format should be jpg. Press the Capture button. Go to the program ImageJ (might need to search for it on the computer) and open your .jpg file. choose 'Rectangular Selection' (the far left box) and then draw a small rectangle around the first set of rings. You want the rectangle at the same vertical height as the center of the ring and it should be pretty narrow. Then Choose Analyze→Plot Profile.

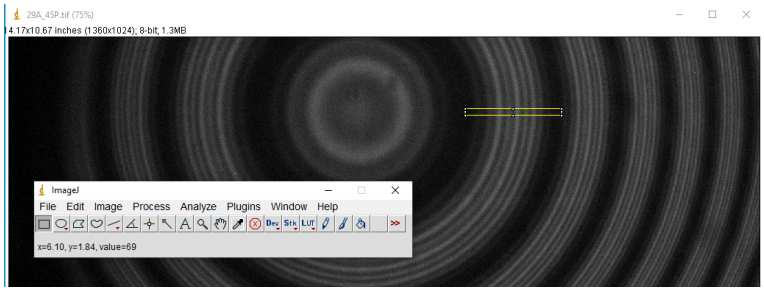

FIG. 4: Example pattern obtained at high field (29 A) and 45° polarization. A rectangular box is selected using ImageJ.

Once you've optimized your signal at high current you should do the following:

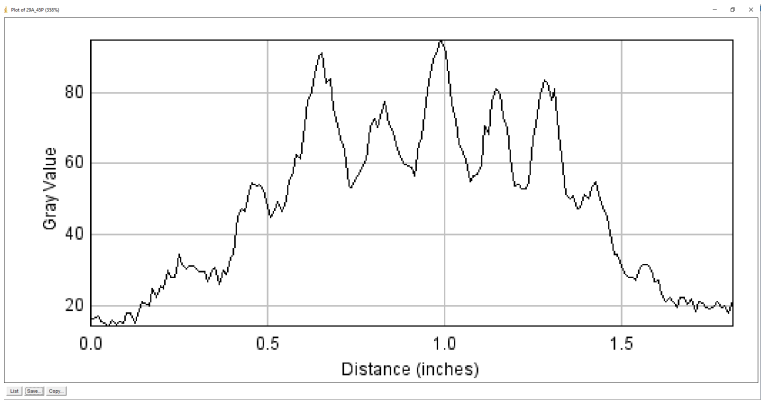

FIG. 5: Plot profile obtained at 29 A and 45° polarization. Nine peaks are visible.

1. If you don't already have a good one, save a capture with the polarizer at 45° as a .jpg file.
2. Move the polarizer to 90°. This is the polarization axis angle with respect to the vertical. So now only horizontally polarized light will pass through the polarizer. Save a capture.
3. Move the polarizer to 0°. So now only vertically polarized light will pass through the polarizer. Save a capture.
4. Keeping the polarizer at 0° lower the current by about 2 A and save a capture. Continue, saving a capture at 0° every 2 A or so to get 12-15 good captures. Finally, turn the current down to 0 A and record one final capture in zero field.

Analyzing data: the green mercury line

The mercury green line at 546.1 nm is a transition from a $^3S_1$ initial state to a $^3P_2$ lower energy state. Determine the g-factor for both of these states. Make a version of Fig. 2 for this transition. The energy splittings in fields should be to scale, i.e. proportional to g. Use selection rules to determine the allowed transitions: there will be nine total transitions. List out these nine transitions with the energy and the ΔM_J value.

A. Using the 0 A data for calibration

IMPORTANT. We are going to be analyzing these images, finding the horizontal location of peaks. Therefore we must have the same origin for each image, yet ImageJ will by default set zero to be the left edge of the box we draw. There are two ways to handle this.

1. Simply always draw boxes starting at the far left edge of the image.
2. As you draw a box the ImageJ app will have a line that says $x = ...$ The $x =$ value is the pixel position of the left edge of the box (pixel for a jpg, inches for a tif file). You will just need to write that down before you finish the box and add this to all otehr numbers. This can improve resolution.

Open up your zero field data in ImageJ. Choose 'Rectangular Section' and draw a box like the one shown in Fig. 6. You'll see that I started teh box at teh far left of the image. It should cover a couple of rings and must capture the inner ring on both the left and right sides. Then Choose Analyze→Plot Profile.

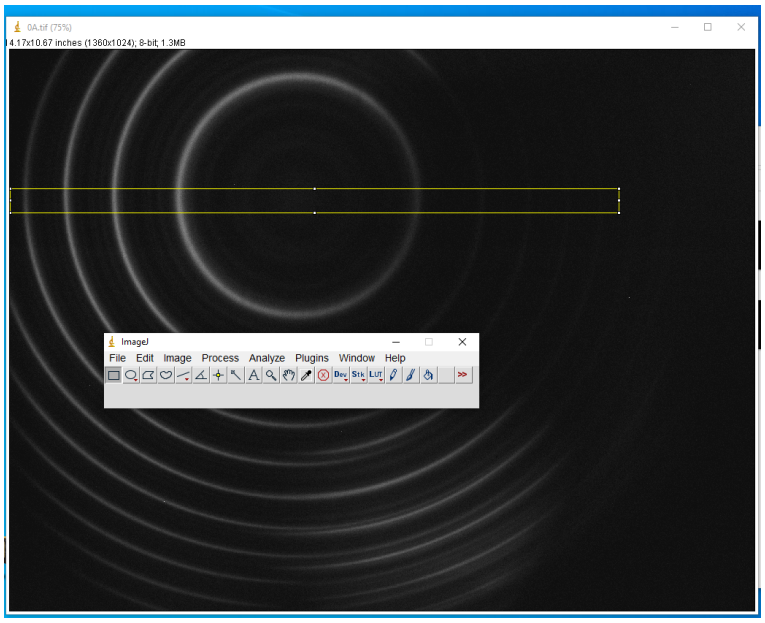

FIG. 6: Example pattern obtained at zero field (0 A). A rectangular box is selected using ImageJ. The box starts at the far left of the image.

You can determine the position of the rings using the cursor tool in ImageJ. We will use this data for our calibration. Remember that this etalon has a free spectral range of 74.9 GHz. That means that shifting the frequency of your light by 74.9 GHz would move each ring to the position of the ring one further out. Also, for small angles the rings are spaced quadratically.

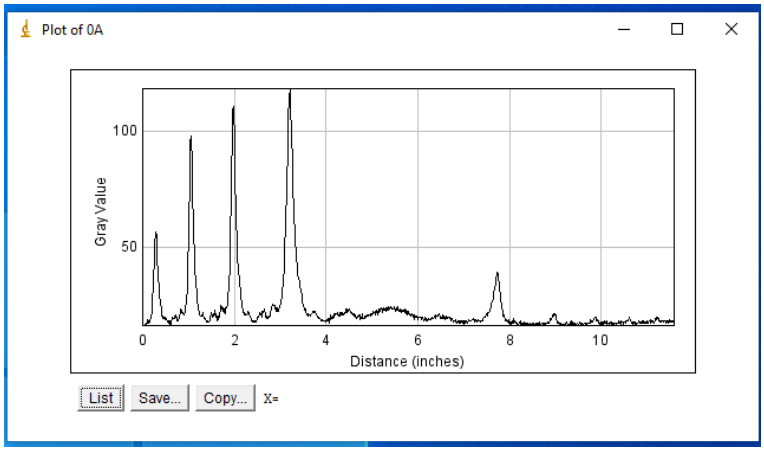

FIG. 7: Example profile plot obtained at zero field (0 A).

Make a plot where the x-axis values are the horizontal positions of your rings. For the y-axis, the innermost ring should be labeled as 0 GHz (on each side), the second ring should be labeled as 74.9 GHz, the third as 2×74.9 GHz=149.8 GHz, etc. Make sure the axes are appropriately labeled with units. Fit this data to a quadratic function.

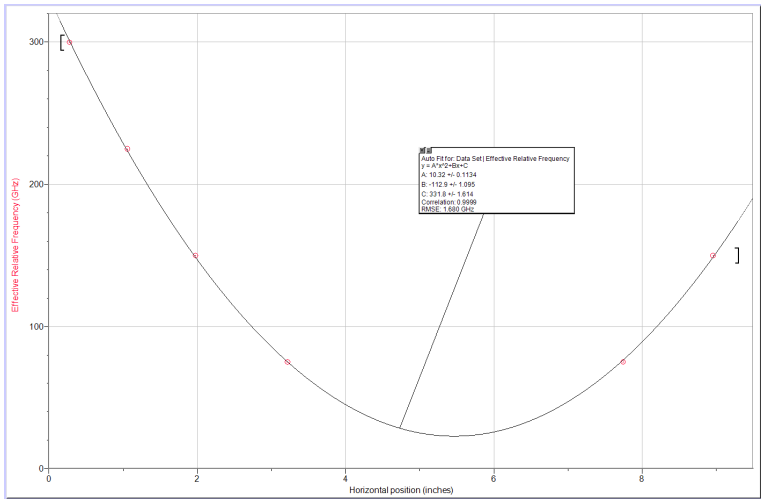

FIG. 8: Example calibration at zero field (0 A).

B. Selection rules

Now use ImageJ to open your highest current data with the polarizer at 45°. Make a another box as before; this box must start at the far left of the image, or else you must add in the position of its edge. Then make a profile plot. Click Copy, and then paste your data into a spreadsheet. THEN USE YOUR QUADRATIC PARAMETERS DETERMINED FROM THE FIT TO CONVERT THIS POSITION DATA

TO FREQUENCY DATA. Determine which set of nine peaks is the best: usually this is from the original first ring, closest to the middle. Since teh middle ring had been defined as zero, this should just be the frequency shift. Copy the data (only the best nine lines) into LoggerPro or Gemini (we don't need a fit here) and make a plot like in Fig. 9 making sure that the axes are labeled appropriately.

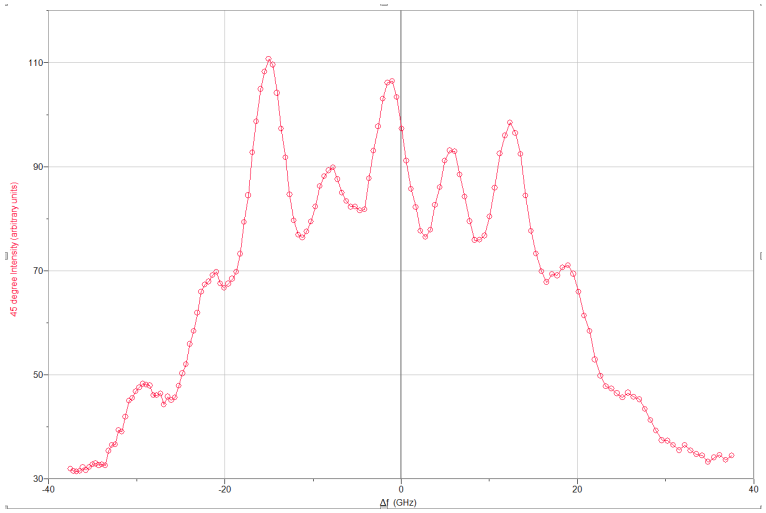

FIG. 9: Example data at a 29 A current through the magnet and the polarizer at 45°.

Do this again for the highest field data where the polarizer was at 0° or 90°. Make similar plots for these data. Determine the ΔM_J selection rules for polarization. What is ΔM_J for the lines that are horizontally polarized (parallel to the applied magnetic field)? What is ΔM_J for the lines that are vertically polarized (perpendicular to the applied magnetic field)?

C. Determining μ_B

You should now follow these steps for each of your 0° polarization measurements. I've suggested using the 0° polarization measurements because that makes the two lines where the energy was $E_0 \pm \mu_B B$ easily discernible. At the very lowest fields (a current of 3 A or less) it might not be possible to tell them apart, but at higher fields it should be.

1. Open the file with ImageJ. Draw a rectangular box like before, making sure that it starts at the far left of the image. Ideally it should cover the innermost ring on both sides of the center.

2. Make a profile plot. Determine the horizontal position of the $E_0 \pm \mu_B B$ peaks. You can do this with the cursor in ImageJ. Usually the innermost ring will be

best to use. If you have good data on both sides of the ring you will end up with four positions: the higher and lower frequency line on each side.

3. Determine the frequency change for each of these four lines.

After doing this, make a figure like that shown in Fig. 10. This shows Δf as a function of current.

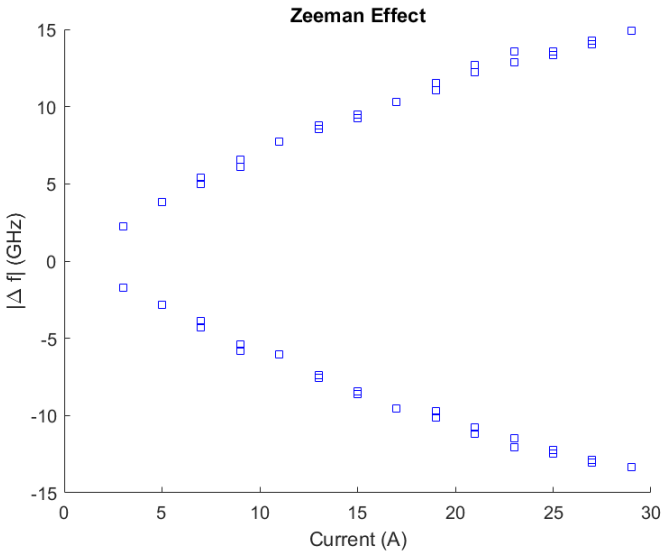

FIG. 10: Example of Δf as a function of current for the $E_0 \pm \mu_B B$ lines.

Finally, use our magnetic field calibration function to turn the current values into magnetic fields. Then plot the absolute value $|\Delta f|$ as a function of magnetic field. Do a fit to the function $y = Ax$ (a line whose y-intercept is fixed to zero). Use this slope fit value (with its uncertainty) to determine an experimental value of μ_B and report this with an uncertainty. In general I suggest keeping one significant figure in your slope uncertainty (unless the digit is a 1 - then keep two sig figs), and enough sig figs in the slope fit value so that it matches the last sig fig in the uncertainty. You can treat this uncertainty in the slope as the only uncertainty when calculating μ_B: you will be multiplying it by values that are either exactly defined constants or are known to high accuracy.

Taking data: the blue mercury line

Now we will take data on the blue line. So we will remove the green filter and replace it with a blue filter. And we will remove the (larger) air-gapped Fabry-Perot etalon (which is well tuned to give sharp line at the green line wavelength) and replace it with the smaller fused silica Fabry-Perot etalon. This etalon is a bit

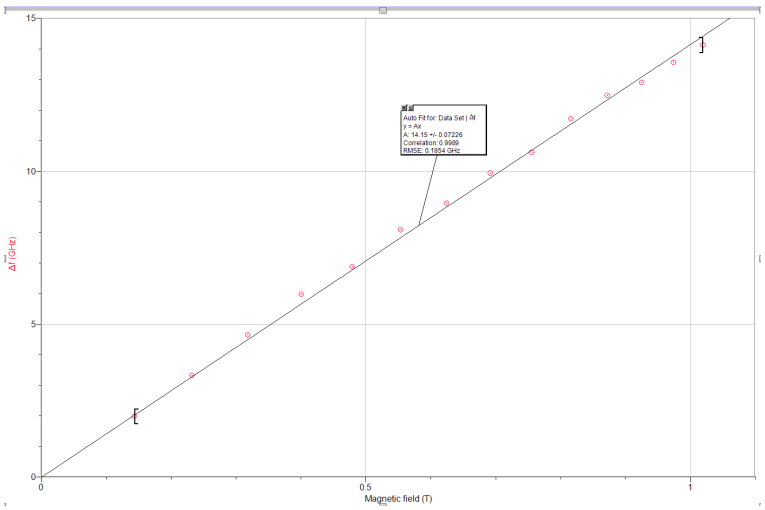

FIG. 11: Example of $|\Delta f|$ as a function of magnetic field for the $E_0 \pm \mu_B B$ lines.

different: its free spectral range is 59.9 GHz and it has a lower *finesse*, which means that the rings will all be a bit broader.

Much like before, start at zero field. Start collecting data and adjust so that several rings are visible on your screen. You can move the lens to focus them, you can rotate the etalon to center the pattern, and you can adjust the camera zoom to make the pattern larger or smaller. The polarizer should start at 45°. Turn the current up to about 20 A: the current should be large enough that all six lines are distinguishable, but not so large that they begin to overlap with lines from the higher or lower order rings (this wasn't much of a concern with the green line measurements, but will be now because the free spectral range is smaller). If needed adjust the lens to make them as sharp as possible (but they won't be quite as sharp as with the green line because of the lower finesse). The do the following:

1. If you don't already have a good one, save a capture with the polarizer at 45° as a .jpg file.
2. Move the polarizer to 90°and save a capture.
3. Move the polarizer to 0° and save a capture.
4. Finally, turn the current down to 0 A and record one final capture in zero field.

Analyzing data: the blue mercury line

The mercury blue line at 435.8 nm is a transition from a $^3S_1$ initial state to a $^3P_1$ lower energy state. Determine the g-factor for both of these states. Make a version of Fig. 2 for this transition. The energy splittings in fields should be to scale, i.e.

proportional to g. Use selection rules to determine the allowed transitions: there will be six total transitions. List out these six transitions with the energy and the ΔM_J value.

Then we are again going to use the zero field data for calibration; the only real difference is that this time the free spectral range is only 59.9 GHz. Open the zero field data and draw a box like in Fig. 6: it should start at the far left of the image, go through the center, and cover at least both sides of the central ring. Make a profile plot. Then use the ImageJ cursor to find the horizontal position of each of these rings. Plot them with horizontal position on the x-axis and effective frequency on the y-axis. This time the innermost ring (both sides) has an effective frequency of 0 GHz, the second ring has 59.9 GHz, the third ring has 119.8 GHz, etc. Fit this data to a quadratic function.

Now use ImageJ to open the data with the polarizer at 45°. Make a another box as before that starts at the far left of the image. Then make a profile plot. Click Copy, and then paste your data into a spreadsheet. Then use the parameters from your quadratic fit to convert this data from position to frequency. Determine which set of six peaks is the best: usually this is from the original first ring, closest to the middle. Copy the data (only the best six lines) into LoggerPro and make a plot like before, making sure that the axes are labelled appropriately. Do this again for the data where the polarizer was at 0° or 90°. Make similar plots for these data. Determine the ΔM_J selection rules for polarization. What is ΔM_J for the lines that are horizontally polarized (parallel to the applied magnetic field)? What is ΔM_J for the lines that are vertically polarized (perpendicular to the applied magnetic field)? Use your current to determine the magnetic field. Determine the Δf for the center of each visible peak and compare this to your expected Δf for each line at this field.