Part I: Newton’s Laws
Chapter 3: Vectors and Coordinate Systems

3.3 Coordinate Systems and Vector Components
3.4 Unit Vectors and Vector Algebra
Vectors with coordinate axes: vector decomposition.

- Let’s adopt a common notation.¹
- Mathematics is a language....
- notation is part of the language.
- Be vigilant!

- \( \vec{A} \) is a vector.
- \( \vec{A}_x \) is a component vector.

¹There is nothing wrong with other notations. Our text sets the notation for this course.
Assessing vector components.

- $\vec{A}_x$ is a component vector.
- $A_x$ is just a component (a scalar, just a number $+$ or $-$ along with units).

Where is not a vector property, but we still need to draw them somewhere!

1. on a dot that represents an object as a particle.
2. on the origin (to facilitate reliable component extraction).
Extracting components.

The magnitude and direction of $\vec{A}$ are found from the components. In this example,

$$A = \sqrt{A_x^2 + A_y^2} \quad \theta = \tan^{-1} \left( \frac{A_y}{A_x} \right)$$

The components of $\vec{A}$ are found from the magnitude and direction.

The angle is defined differently. In this example, the magnitude and direction are

$$B = \sqrt{B_x^2 + B_y^2} \quad \phi = \tan^{-1} \left( \frac{B_y}{|B_y|} \right)$$

Minus signs must be inserted manually, depending on the vector's direction.

Work with angles between 0 and 90° to avoid sign errors:

- sine and cosine are both + in the 0 to 90° range.
- We make a call on sign separately.
Seeing the components in the vectors: unit vectors.

The unit vectors have magnitude 1, no units, and point in the $+x$-direction and $+y$-direction.

We say

- “i hat” for $\hat{i}$.
- “j hat” for $\hat{j}$.

Unit vectors identify the $x$- and $y$-directions. Vector $A_x \hat{i}$ has length $A_x$ and points in the direction of $\hat{i}$. 
Keep your axes tilted.²

▷ Your world is real.
▷ Your axes are a tool.

▷ Keep your view of the world intact.
▷ Get adept with your tools.

The components of \( \vec{C} \) are found with respect to the tilted axes.

\[
\vec{C} = C_x \hat{i} + C_y \hat{j}
\]

Unit vectors \( \hat{i} \) and \( \hat{j} \) define the x- and y-axes.

²There are often good reasons to use tilted axes, but downward vectors should always point down!
For you to do: Let’s try out tilted axes.

- Draw axes like the on the previous slide.
- Take your $x$-axis to be tilted $30^\circ$ above horizontal.
- $\vec{A} = (100 \text{ m/s, downward})$.
- Draw $\vec{A}$ and its decomposition. Label all vectors.
- Write in angles (be careful!) to aid with choosing trig functions.
- Compute the components $A_x$ and $A_y$.

Express these vectors using unit vectors and your computed components:

- $\vec{A}_x$
- $\vec{A}_y$
- $\vec{A}$