Part II: Conservation Laws
Chapter 10: Interactions and Potential Energy

10.5 Energy Diagrams
10.6 Force and Potential Energy
10.7 Conservative and Nonconservative Forces
Energy is transferred to (and from) the system.

Energy is transformed within the system.

System

\[ \Delta E_{\text{sys}} = W_{\text{ext}} \]

Environment

Work done on system \( W_{\text{ext}} > 0 \)

Work done by system \( W_{\text{ext}} < 0 \)
What if we began with $U(x,y,z)$? Can we extract $\vec{F}(x,y,z)$?

In many scientific and engineering contexts, it is more common to be given $U(x,y,z)$ as a way of describing an interaction.

Recall: $\Delta U \equiv -W$ where $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$.

$\Delta U \equiv - \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$.

We can go in reverse: $\vec{F} = -\nabla U$.

“The force is the negative of the gradient of the potential energy.”

By component: $F_x = -\frac{\partial U}{\partial x}$, $F_y = -\frac{\partial U}{\partial y}$, $F_z = -\frac{\partial U}{\partial z}$.

In 1D: $\Delta U \equiv - \int_{x_i}^{x_f} F_x dx$, $F_x = -\frac{dU}{dx}$. 
It’s not so bad! Start by confirming gravity and springs.

- $U_G = mgy$
- $\vec{F}_G = (-mg) \hat{j}$
  
  $- \frac{d}{dy}(mgy) = -mg$

- $U_{\text{Spring}} = \frac{1}{2}kx^2$ (equilibrium at $x = 0$).
- $\vec{F} = (-kx) \hat{i}$
  
  $- \frac{d}{dx}(\frac{1}{2}kx^2) = -kx$
For you to do: Consider the following potential energy function in 1D along $x$,

$$U(x) = A \sin \left( \frac{2\pi x}{B} \right).$$

- Carefully draw $U(x)$ making sure to work in $A$ and $B$.

- Label the places where $F_x$ is zero.
  - Which are points of **stable equilibrium** (SPRING-LIKE force in local vicinity)?
  - Which are points of **unstable equilibrium** (RUN-AWAY force in local vicinity)?

- Where is the force largest in magnitude? What is this largest magnitude?
  - For which of these locations is $F_x$ positive?
  - For which of these locations is $F_x$ negative?