Part III: Applications of Newtonian Mechanics
Chapter 12: Rotation of a Rigid Body

12.3 Rotational Energy (continued)
12.4 Calculating Moment of Inertia
\[ I = \int r^2 \, dm \]

\[ K_{\text{rot}} = \frac{1}{2} I \omega^2 \]

**TABLE 12.2** Moments of inertia of objects with uniform density

<table>
<thead>
<tr>
<th>Object and axis</th>
<th>Picture</th>
<th>( I )</th>
<th>Object and axis</th>
<th>Picture</th>
<th>( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin rod, about center</td>
<td><img src="image" alt="Thin rod" /></td>
<td>( \frac{1}{12} ML^2 )</td>
<td>Cylinder or disk, about center</td>
<td><img src="image" alt="Cylinder or disk" /></td>
<td>( \frac{1}{2} MR^2 )</td>
</tr>
<tr>
<td>Thin rod, about end</td>
<td><img src="image" alt="Thin rod" /></td>
<td>( \frac{1}{3} ML^2 )</td>
<td>Cylindrical hoop, about center</td>
<td><img src="image" alt="Cylindrical hoop" /></td>
<td>( MR^2 )</td>
</tr>
<tr>
<td>Plane or slab, about center</td>
<td><img src="image" alt="Plane or slab" /></td>
<td>( \frac{1}{12} Ma^2 )</td>
<td>Solid sphere, about diameter</td>
<td><img src="image" alt="Solid sphere" /></td>
<td>( \frac{2}{5} MR^2 )</td>
</tr>
<tr>
<td>Plane or slab, about edge</td>
<td><img src="image" alt="Plane or slab" /></td>
<td>( \frac{1}{3} Ma^2 )</td>
<td>Spherical shell, about diameter</td>
<td><img src="image" alt="Spherical shell" /></td>
<td>( \frac{2}{3} MR^2 )</td>
</tr>
</tbody>
</table>
Two ways we encounter rotational energy.

**Fixed axle:** rotation about the axle, that tells the entire story (just $\omega$).

- $K = K_{\text{rot}}$
- $K = \frac{1}{2}I\omega^2$

**Unconstrained:** rotational and linear motion combined ($\omega$ and $v_{\text{cm}}$). Examples:

1. A thrown object’s center of mass arcs through space like a projectile while the object spins around its center of mass.
2. A bowling bowl early in its path down the lane. The spinning of the ball is decoupled from it’s linear motion (kinetic friction).
3. A bowling ball late in its path: rolling without slipping (static friction, $v_{\text{cm}} = r\omega$).

- $K = K_{\text{rot}} + K_{\text{cm}}$
- $K = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2$
Disk about center

\[ I = \int (r^2 \ast dm) \]

\[ I = \int_0^R \left( r^2 \ast M \left( \frac{2\pi r \ast dr}{\pi R^2} \right) \right) \]

\[ I = \frac{2M}{R^2} \int_0^R r^3 \, dr \]

\[ I = \frac{2M}{R^2} \ast \frac{R^4}{4} \]

\[ I = \frac{1}{2} MR^2 \]
For you to do:

1. Work through this to show that $I = \frac{1}{3}ML^2$ for a thin rod about its end.

2. Alter and rework the integral to show $I = \frac{1}{12}ML^2$ for a thin rod about its center.

3. Show a connection between these results and the parallel axis theorem,

$$I_{\text{off-center axis}} = I_{\text{cm}} + Md^2 = Md^2 + I_{\text{cm}}$$

where $d$ is the (center of mass)$\rightarrow$(off-center axis) distance.

4. Also show a connection between these results by viewing the “about the center” case as two “half rods” about their ends.