Reply to Comment on ‘Coefficient of performance of Stirling refrigerators’

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Received 11 December 2019, revised 15 May 2020
Accepted for publication 17 June 2020
Published 17 August 2020

Keywords: regenerator, thermodynamic system, equilibrium, coefficient of performance, Stirling refrigerator

Dickerson and Mottmann raise four pedagogical issues in their comment: (a) the idealization of regenerators in section 2, (b) the choice of system in section 3, (c) the nature of equilibrium in section 4, and (d) the calculation of the coefficient of performance (COP) in section 5. These topics are addressed in order in the following paragraphs.

(a) A perfect regenerator does not exist in the real world. Classical thermodynamics treats idealizations such as ideal gases, adiabatic expansions, equilibrium states, quasi-static processes, and Carnot cycles as limits that can be approached or approximated. Likewise, the complexity of a regenerator is in the design details and not in the fundamental concept, which is that of a unit that can store thermal energy to be returned later on demand. It is not necessary to consider the engineering aspects of real regenerators in a physics course, as the basic idea suffices to enrich thinking about thermodynamic devices. Specifically, there are several reasons to introduce a regenerator into the discussion of refrigerator design: (i) because it can improve the COP; (ii) because real Stirling refrigerators incorporate regenerators; and (iii) because it is an innovative thermodynamic device worth knowing about. One might compare it to a commutator, which is typically introduced in textbook discussions of a dc motor. Both a regenerator and a commutator are secondary to the main theoretical principles. Yet their inclusion is helpful because they overcome important limitations in what machines can otherwise achieve.

(b) Delineation of the system from the surroundings is crucial in analyzing the energetics of any process or device in physics [1]. Different choices of the system boundaries lead to different equations and ways of thinking about how energy flows into, flows out of, and is stored within a system. Including the regenerator within the system when considering the thermodynamics of a Stirling refrigerator is a natural choice when one is interested in the overall wall-plug COP of the device.

(c) Consider dividing the working fluid flowing inside a Stirling refrigerator into individual parcels or subsystems, each in a different part of the apparatus or a different layer of the
regenerator. It is not necessary to treat the entire fluid as a single equilibrium volume. That would imply the whole gas must move as a block through each successive point in the cycle (infinitely slowly to avoid irreversibilities). In an actual fridge, the coolant fluid fills a closed circulating loop. Subvolumes of the gas each experience different thermodynamic processes, treated separately so that each parcel is in its own near-equilibrium state [2]. In other words, partition the gas into spatially isolated portions distributed around the loop that are all individually equilibrated¹. That is not conceptually harder to imagine than the entire gas as a unit moving in a series of isolated time intervals through the various stages of the cycle.

(d) Dickerson and Mottmann ask how one determines what part of the energy transfers in a cycle is ‘wanted’. One might also ask how much is ‘paid’. These two terms are colloquialisms. The ‘payment’ refers to the electrical cost (averaged over any integer number of cycles) to run the refrigerator for a given cooling energy. If one can store some of the thermal energy output during one portion of the cycle and return it as input later in another portion of the cycle, then that avoids wasteful irreversibilities associated with the alternative of transferring heat to or from the hot and cold reservoirs during the isochoric steps. Regarding the ‘wanted’ part of the heat flow, only the heat extracted while the coolant gas is in thermal contact with the sample during stage cd contributes to the useful cooling power. The heat withdrawn from the regenerator during stage da does not cool the sample and thus it should not be included in the performance calculation. Defining the COP as \(Q_{cd} + Q_{da}/W\) rather than as \(Q_{cd}/W\) would artificially increase its value². Not all heat inputs to the ideal gas are to be included in the numerator of the COP calculation. Instead the value of \(Q_C\) is defined according to the application. This idea is reflected in Leff’s suggestion [3] that the efficiency (or in the present case the COP) be qualified with the adjective ‘task-specific’. It lies behind the assertion [4] that the COP needs to be independent of ‘how the system boundaries’ are chosen: whether the regenerator is placed inside or outside the system cannot change the value of the COP of a given device. To put it another way, \(Q_C\) does not refer to ‘heat input to the system’ (which would depend on its boundaries) but instead to ‘heat removed from the cooling compartment’ (which does not change when the system boundaries are changed).

In closing, a reversible Stirling cooler with a regenerator has a COP equal to that of a reversible Carnot cooler operating between the same hot and cold reservoirs. The same is possible for other cycles such as an Ericsson cooler. The important points are that these counter-clockwise cycles must be reversible and they must be connected to only two external temperature reservoirs (and thus a device such as a regenerator must take care of any other heat transfers besides \(Q_C\) and \(Q_H\)). The Carnot cycle is not special in being the only fridge to have this COP. It is special because it is the only fridge with this COP that does not require a regenerator. It thus retains its special status in physics teaching as the simplest possible refrigerator that attains the maximum COP for given hot and cold reservoir temperatures.

Thanks to Harvey S Leff for suggestions that improved this reply.

¹ Each subvolume of the gas can contain the same number of molecules, chosen to be small enough that each portion can be plotted as a point on a PV diagram but large enough to be in the thermodynamic limit. The entire set of points then encompasses the whole gas and traces out the entire cycle at a single instant in time.

² The value of W is the net work done by the whole cycle, regardless of whether \(Q_C\) includes \(Q_{da}\) or not. There is no compensating change in the value of W if one changes the definition of \(Q_C\).
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References

Comment

Comment on ‘Coefficient of performance of Stirling refrigerators’

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Received 9 November 2019, revised 2 May 2020
Accepted for publication 17 June 2020
Published 17 August 2020

Abstract
This comment examines an article that develops a counterclockwise reversible Stirling cycle whose coefficient of performance equals that of a Carnot refrigerator. Unfortunately, among other issues, the proposed cycle is never in thermodynamic equilibrium and thus cannot match an idealized reversible Carnot.

Keywords: thermodynamics, refrigerators, Stirling cycle

1. Introduction

The present authors have shown [1] that clockwise (CW) Stirling cycles, even if reversible, have efficiencies ($\varepsilon$) that are strictly less than that of an ideal Carnot cycle, $\varepsilon_{\text{Stirling}} < \varepsilon_{\text{Carnot}}$. We have also published ‘Not all counterclockwise thermodynamic cycles are refrigerators’ [2, 3], which showed that counterclockwise (CCW) Stirling cycles could not act as refrigerators. Recently, Professor Mungan described [4] a reversible CCW Stirling cycle with the claim that it is a refrigerator and has a coefficient of performance (COP) equal to an ideal Carnot refrigerator. The purpose of this comment is to show that the claim is unfortunately in doubt.

The Introduction in [4] describes an idealized CCW Stirling cycle that ‘undergoes two isothermal and two isochoric processes . . . these processes are here assumed to be quasistatic (so that they can be graphed on a pressure–volume diagram)’. The pressure–volume ($PV$) diagram of such an idealized Stirling cycle is shown in figure 1.

Measurements of cylinder pressures and volumes for functioning Stirling engines are available. An entertaining example, and plotted in real time, can be seen on the internet [5]. About one minute into the video an external load is removed from the engine and the shape of the curve changes. Stirling engines incorporate two connected cylinders, each with its own moveable piston. Figure 2 is a simplified version of a diagram originally from reference [6]. The separate curves are the pressures and volumes measured for each of the two interconnected cylinders.
Figure 1. The idealized CCW Stirling cycle. The cycle operates with isochors and isotherms (dotted) at temperatures $\tau_{\text{cold}}$ and $\tau_{\text{hot}}$. Open arrows indicate heat directions into or out of the ideal gas system for each of the four processes.

Figure 2. A $PV$ diagram for each of the two cylinders of a single working Stirling engine. This drawing is based on a similar diagram in reference [6]. Note the extreme difference between these measurements of the two cylinders of a real engine and that of a $PV$ diagram of an idealized Stirling cycle.

The performance of a real device, as seen in figure 2 and the video, has little resemblance to the idealized $PV$ diagram of figure 1. These examples should help dispel any notion that real Stirling engines have anything in common with idealized theoretical cycles. This is not a surprise: real devices can never be in thermodynamic equilibrium. The surprise is that real Stirling devices and idealized Stirling cycles are so often commingled. That is a concern about reference [4]; it is supposedly about quasi-static processes yet its introduction gives eight consecutive references about functioning Stirling engines.
2. Regenerators

The Reverend Robert Stirling’s original engine included what he called an ‘economizer’ and is today referred to as a regenerator; it is simply a heat exchanger. Real regenerators do exist as part of some real Stirling engines. Reference [4] correctly notes that a reversible Stirling cycle without a regenerator cannot match the performance of a Carnot. Of course a real regenerator (commonly consisting of a metal wire mesh) is never in thermodynamic equilibrium and thus much effort has gone into developing an idealized regenerator to allow for simplified analysis.

The idealized regenerator is ‘composed of a set of thermal layers isolated from each other and arranged along the flow axis’ [4]. Designs exist [7, 8] for such a regenerator and figure 3 provides a sketch of a diagram from reference [8]. Figure 3(a) shows a cross-section of the device. Two cylinders with their pistons are in contact with external heat reservoirs at temperatures $\tau_{\text{cold}}$ and $\tau_{\text{hot}}$. The ideal gas in the two cylinders is continuously interconnected through the regenerator. The thermal layers are represented by vanes within the regenerator. Pistons force the ideal gas to flow past the vanes whose temperatures vary linearly with position inside the regenerator.

Figure 3(b) is a plot of $T$, the temperature of the ideal gas working substance. The temperature $T$ varies smoothly within the regenerator because the number of vanes is assumed to be infinite and thus each vane differs in temperature by only $d\tau$ from its neighbor.

To achieve the desired outcome requires an idealized but complex regenerator: ‘the heat capacity of the layer is equal to that of the gas, 2.5 $nR$, by appropriate choice of the thickness of the layer. Since the cross-sectional area $A$ of the layer is an independent parameter, this choice of thickness does not restrict the specific heat $c$ of the layer, i.e. $c$ can be made large by choosing the regenerator to have small $A$. A small cross-section also makes it easier to establish good thermal contact between the gas and the entire layer’ [4]. Within the regenerator
it is also required that ‘every layer can hold its temperature’ as gas continuously flows through it.

The presumption is that with these conditions the entire system is reversible and can be analyzed in the manner familiar from classical theoretical thermodynamics. With the regenerator constructed as described, the result is a CCW Stirling device that functions as a refrigerator with a COP equal to that of a Carnot refrigerator.

3. Reality

Why is such a complex regenerator necessary for an otherwise elementary thermodynamics problem? The ultimate answer is to ensure the cycle matches a Carnot cycle. A Carnot cycle consists of isotherms and adiabats; a Stirling also employs isotherms but has isochors. To convert a Stirling into a Carnot therefore requires the elimination of the isochors. The regenerator’s stated purpose is to absorb/eject the isochoric heats $Q_{bc}$ and $Q_{da}$ (figure 1). But by itself this regenerator is not enough.

The conversion of a Stirling cycle into a Carnot necessitates the introduction of a unique ‘system’ consisting of both the ideal gas and the regenerator. Under these conditions the isochoric heats $Q_{bc}$ and $Q_{da}$ never leave the newly defined ‘system’. Heats $Q_{bc}$ and $Q_{da}$ are now ignored since by definition they are purely internal. Because $Q_{bc}$ and $Q_{da}$ have no external effect the isochors can be imagined to be $Q = 0$ ‘adiabats’. Removing isochors and replacing them with adiabats makes the original cycle a Carnot and it is no surprise that the cycle now has a Carnot’s COP.

Reference [4] begins by mentioning idealized quasi-static processes. But the paper also discusses vane temperature, vane heat capacity, vane specific heat $c$, vane area $A$, and vane thickness. The regenerator is far removed from familiar theoretical thermodynamics that employs ideal gases contained within a single volume $V$. We question how this carefully designed regenerator can be considered part of idealized classical thermodynamics.

4. Thermal equilibrium

There is a crucial and fundamental physics problem with the device sketched in figure 3. That problem is clearly exhibited in the graph of figure 3(b). The graph does not show the variation of the ideal gas temperature at different stages of the cycle. Instead the figure plots the permanent variation of $T$ within the device itself. The ideal gas never has a single temperature; $T$ always varies throughout volume $V$.

It is required that within the regenerator ‘every layer can hold its temperature’ [4]. But the ideal gas cannot be in thermodynamic equilibrium if it has embedded within it a regenerator with its permanent distribution of temperatures. In addition, from figure 3(a), it is seen that some ideal gas is always in contact with the cold reservoir at temperature $τ_{\text{cold}}$ while simultaneously some ideal gas is in contact with the $τ_{\text{hot}}$ hot reservoir.

‘A quasi-static process is defined as a succession of equilibrium states’ that do ‘not involve considerations of rates, velocities, or time. The quasi-static process simply is an ordered succession of (equilibrium) states’ [9]. At each step in a quasi-static process the system must be in equilibrium. Only then are its thermodynamic properties such as $P$, $V$, and $T$ well defined throughout the entire system. Since the Stirling device described in [4] is never in equilibrium it is obvious that it cannot be depicted in a $PV$ diagram, thus violating the original requirement that ‘these processes are here assumed to be quasistatic (so that they can be graphed on a pressure–volume diagram)’ [4].
The Stirling device, so carefully developed and explored in the article under review, is never in equilibrium. Consequently it simply cannot match the performance of an idealized reversible Carnot cycle, much less even be a quasi-static reversible cycle.

5. **Q** that is ‘wanted’

A \(PV\) diagram gives all the information that can be known about a cycle: heat \(Q_{\text{in}}\) enters the ideal gas and heat \(Q_{\text{out}}\) leaves. Each contribution to \(Q_{\text{in}}\) and \(Q_{\text{out}}\) must be consistently included, and every cycle’s COP must be determined in the same consistent way: \(\text{COP} = \frac{Q_{\text{out}}}{Q_{\text{in}} - Q_{\text{out}}}\). For the Stirling cycle (figure 1) we see that \(Q_{\text{in}} = Q_{\text{cd}} + Q_{\text{da}}\), and \(Q_{\text{out}} = Q_{\text{ab}} + Q_{\text{bc}}\). Yet for the Stirling cycle, and only the Stirling, it seems permissible to ignore parts of \(Q_{\text{in}}\) and \(Q_{\text{out}}\) because they are deemed to be ‘internal’.

We agree with reference [4]: ‘If the COP depended on how the system boundaries were chosen, then the COP would cease to represent an objective figure of merit’. Reference [4] tests this by briefly using the cycle’s ideal gas as the true system, not the previous ‘system’ with regenerator. In other words, \(Q_{\text{da}}\) and \(Q_{\text{bc}}\) are no longer claimed to be internal and thus no longer ignored. But this radical change in system boundaries is declared to be irrelevant: \(Q_{\text{cd}}\) alone should be included since it ‘represents the energy transfer wanted’ [4]. Using only the ‘wanted’ heat \(Q_{\text{cd}}\) again results in a cycle with a Carnot COP. The result of using ‘wanted’ \(Q\) is taken as confirmation of the previous regenerator analysis.

But what is a ‘wanted’ \(Q\)? Unfortunately this is not explained. Selecting only ‘wanted’ \(Q\) basically means that almost any result can be obtained. We are unaware of any other idealized cycle whose analysis is based on ‘wanted’. The concept of ‘wanted’ has little validity and cannot be used to justify a regenerator that is never in thermodynamic equilibrium.

6. **Final considerations**

The performance of the Carnot cycle is a consequence of the 2nd law of thermodynamics. This maximum performance ‘holds even if the gas is not ideal, and, for that matter, even if the working substance is not a gas at all’ [7, p 125]. Chemical, magnetic, or electrical systems all have the same performance when taken through a suitable reversible Carnot cycle. The Carnot represents truly fundamental physics.

Yet in reference [4] a long list of special conditions is necessary to achieve a desired result: ‘the heat capacity of the layer … the thickness of the layer … the cross-sectional area \(A\) of the layer … the specific heat \(c\) of the layer … good thermal contact’. It is difficult to believe that such a detailed regenerator, never in thermodynamic equilibrium, can compete with a Carnot cycle whose foundation rests on the 2nd law.

Finally, note that the refrigerator coefficient of performance is defined only for cycles that are actually capable of refrigeration. It is true that all CW cycles are viable heat engines, nevertheless ‘not all counterclockwise thermodynamic cycles are refrigerators’ [2]. The CCW Stirling cycle violates all three criteria established in [2] that are necessary for a viable refrigerator. Thus it is necessary to stop here because it is meaningless to attempt to compute a COP for a cycle that is not a viable refrigerator.

7. **Conclusions**

The author Professor Mungan has meticulously described a Stirling cycle regenerator in order to create a reversible refrigerator. The regenerator’s specific features are complex and perhaps
unique in elementary classical thermodynamics. Thus the described regenerator presents no
general insights applicable to the field of idealized theoretical thermodynamics.

The regenerator is carefully designed to obtain a single result: the Stirling cycle matches
the Carnot cycle. But the Stirling cycle, using the regenerator described in [4], is never in
thermodynamic equilibrium and thus cannot equal the coefficient of performance of a
reversible Carnot refrigerator. In fact, it is not a quasi-static cycle and thus cannot even be
plotted on a $PV$ diagram.

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