



PAPER

Comparison of series and parallel undriven *RLC* circuits

Carl E Mungan*

Volgenau Department of Physics, US Naval Academy, Annapolis, MD 21402, United States of America

* Author to whom any correspondence should be addressed.

E-mail: mungan@usna.edu**Keywords:** *RLC* circuit, time constant, underdamped oscillations, Kirchhoff's rules, quality factor, oscilloscope measurements

RECEIVED

12 July 2026

REVISED

4 September 2026

ACCEPTED FOR PUBLICATION

9 September 2026

PUBLISHED

23 September 2026

Abstract

The undriven series *RLC* circuit is typically presented in the second semester of an introductory physics course as a key example of free decay, analogous to the motion of a mass on a spring in the presence of drag that was discussed in the first semester. An advantage an electrical circuit has over a mechanical setup is that it is easy to graphically display the behavior on an oscilloscope. If the resistance is sufficiently small, then underdamped oscillations are observed on the screen, and students can vary the resistance, inductance, and capacitance to demonstrate how they affect the frequency and damping of the oscillations. As an extension of the familiar series circuit, here it is explored both theoretically and experimentally what happens if a resistor is placed in parallel with the capacitor, either in place of or in addition to the resistor in series with the inductor. (However, a parallel *RLC* circuit assumes a lossless inductor with zero series resistance and thus it cannot be realized experimentally, although it can be modeled theoretically. Consequently, the measurements reported here are for a series and a combined parallel-series circuit, but not for a parallel circuit.) This extension illustrates similarities and differences between parallel and series connections, without introducing significant complexities in the physics or mathematics. The ideas therefore remain accessible to introductory physics majors.

1. Introduction

The study of electrical circuits is important because modern technology relies on electronic devices. They are the foundation of power management, communications, computers, and medical instrumentation. Additionally, electronics are at the heart of many modern tools. Experimental work requires measuring instruments which typically use electrical circuitry. Such circuits, in turn, are built upon the concepts of capacitance, inductance, and resistance.

Given a capacitor *C*, an inductor *L*, and a resistor *R*, pairs of them can be wired together to form three different kinds of circuits [1], each having a response with its own characteristic time parameter (assuming a DC power supply is switched in and then out of the circuit to initially energize it). Namely, connecting together the capacitor and inductor results in sinusoidal oscillations of the current with a period of

$$\frac{2\pi}{\omega} = 2\pi\sqrt{LC} \quad (1)$$

where ω is the angular frequency. Wiring together the capacitor and resistor gives rise to an exponentially decaying current with a 'capacitive' time constant of

$$\tau_C = RC. \quad (2)$$

Finally, if the leads of the inductor and resistor are connected, the current again decays exponentially but with an 'inductive' time constant of

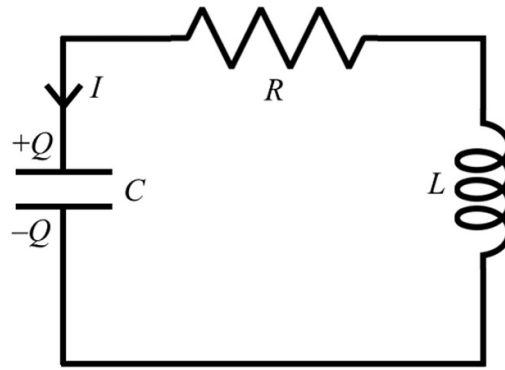


Figure 1. An undriven series *RLC* circuit. The time-dependent charges on the capacitor plates are $\pm Q$ and the current to all elements is I .

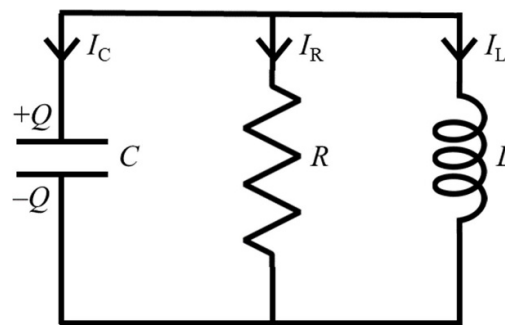


Figure 2. An undriven parallel *RLC* circuit. The time-dependent charges on the capacitor plates are $\pm Q$ and the currents I in the three branches are each subscripted with the element in that branch.

$$\tau_L = \frac{L}{R}. \quad (3)$$

Now suppose all three elements are wired together. That can be done in two different ways, either in series as shown in figure 1 or in parallel as shown in figure 2. The step-energized *undriven* parallel *RLC* circuit in figure 2 is not discussed in standard introductory physics textbooks, although some of them [1–3] treat the sinusoidally *driven* parallel *RLC* circuit. The primary purpose of this article is to systematically compare the properties of the circuits in figures 1 and 2, with an eye toward understanding how they depend on the circuit parameters. Additionally, a more realistic circuit containing both parallel and series resistances is investigated theoretically and experimentally. In contrast to physics textbooks, introductory electrical engineering textbooks [4–7] frequently analyze the response of a parallel *RLC* circuit to a step excitation, but without probing into the details of the expressions involved. In other words, they are generally content to present the equations and their mathematical (or computer) solutions, without pausing to consider pedagogical issues such as why certain constants do or do not appear in them, or how the parameters could be rewritten in other ways to shed additional light upon them. (In all fairness, engineering texts instead focus on treating a greater *variety* of circuits than do physics texts, and in presenting more *applications* of such circuits.) Physics students could be given aspects of parallel *RLC* circuits to analyze in homework or lab, to practice and extend previously presented ideas for the series version.

2. Theory

In figures 1 and 2, a different current has been assigned to each branch of the circuit. Specifically, there is only one branch and hence only one current I in figure 1. The arrow indicating the direction of that current has been chosen to point toward the plate of the capacitor that is assumed to have positive charge $+Q$ so that a positive current I increases Q and thus $I = +\dot{Q}$ (where the overdot denotes a time derivative). Likewise, in figure 2 where there are three branches (and the currents in them have been

subscripted with the circuit element toward which each current is directed) the direction of I_C has been chosen to give a positive time derivative, $I_C = +\dot{Q}$, of the charge on the capacitor.

Both circuits are analyzed using Kirchhoff's current junction and voltage loop rules. These two rules imply the following facts about figures 1 and 2. In the series circuit, the *same* current flows in and out of the leads of the inductor, resistor, and capacitor, and the *sum* of the voltages across these circuit elements (all measured in the same direction around the loop) is zero. In the parallel circuit, the *same* voltage exists across the inductor, resistor, and capacitor (all measured up the page), and the *sum* of the currents flowing out of the top junction is zero¹. One might remember these facts by the phrases 'in series: same current and add voltages' and 'in parallel: same voltage and add currents' which, for example, can be used to derive the equivalent resistance of resistors in series or parallel (and likewise the equivalent capacitance or inductance of capacitors or inductors in series or parallel).

In particular, beginning at the lower left corner of the circuit in figure 1 and proceeding clockwise around the loop, the sum of the voltages across the three elements is

$$\frac{Q}{C} + IR + L\dot{I} = 0. \quad (4)$$

Divide this equation by L and substitute $I = \dot{Q}$ into it to obtain a second-order linear homogeneous differential equation for dependent variable Q as a function of the independent variable t ,

$$\ddot{Q} + \frac{\dot{Q}}{\tau_L} + \omega^2 Q = 0 \quad (5)$$

using the quantities defined in equations (1) and (3). Every term in this equation has units of charge divided by time squared.

It takes only a little extra work to get the differential equation for the circuit in figure 2. The voltage across the capacitor equals that across the resistor (measuring both voltages upward) so that

$$\frac{Q}{C} = I_R R \Rightarrow \dot{I}_R = \frac{\dot{Q}}{RC}. \quad (6)$$

Likewise, the voltage across the capacitor equals that across the inductor,

$$\frac{Q}{C} = L\dot{I}_L \Rightarrow \dot{I}_L = \frac{Q}{LC}. \quad (7)$$

Lastly, the time derivative of the current to the capacitor is

$$I_C = \dot{Q} \Rightarrow \dot{I}_C = \ddot{Q}. \quad (8)$$

The sum of the currents is zero for this parallel circuit,

$$I_C + I_R + I_L = 0 \Rightarrow \dot{I}_C + \dot{I}_R + \dot{I}_L = 0. \quad (9)$$

Substitute equations (6) to (8) into (9) to obtain

$$\ddot{Q} + \frac{\dot{Q}}{\tau_C} + \omega^2 Q = 0 \quad (10)$$

using the quantities defined in equations (1) and (2).

Remarkably, equations (5) and (10) have exactly the same form. The only difference is one equation uses the time constant from equation (2), whereas the other uses the time constant from equation (3). That realization leads to the question: Why does the series circuit use the inductive time constant and the parallel circuit use the capacitive time constant? It is reasonable that ω , τ_L , and τ_C appear in the differential equations, because the two circuits contain a resistor, inductor, and capacitor. But why does only *one* of the time constants appear in each equation instead of *both* time constants?

¹ Some of the currents and voltages must therefore be negative. A negative current means it is flowing in the direction opposite to its indicated arrow. A negative voltage means the polarity across that element is opposite to what has been assigned to it (according to the charges on the capacitor plates for example) or to the polarity of the measuring device (according to the placement of the positive and negative leads of a voltmeter for example).

The surprising answer is: Both do in fact! To see that, divide equation (5) by ω to get

$$\frac{\ddot{Q}}{\omega} + \frac{\dot{Q}}{\omega\tau_L} + \omega Q = 0. \quad (11)$$

The coefficient of $\dot{Q}$ in this equation is the reciprocal of $\omega\tau_L$ which can be rewritten in some revealing alternative forms,

$$\omega\tau_L = \frac{1}{\omega\tau_C} = \sqrt{\frac{\tau_L}{\tau_C}} = \frac{X}{R} = \Gamma_{\text{series}}. \quad (12)$$

Here $X = (L/C)^{1/2}$ is the characteristic impedance (in ohms) of an LC circuit, and $\Gamma_{\text{series}} = X/R$ is the quality factor² of an RLC series circuit. In figure 1, the voltage drop across the resistor will be small (so that the resistor only weakly dampens the oscillations of the LC circuit) if $R \ll X$, corresponding to a high quality factor.

In like fashion, dividing equation (10) by ω results in

$$\frac{\ddot{Q}}{\omega} + \frac{\dot{Q}}{\omega\tau_C} + \omega Q = 0 \quad (13)$$

and the reciprocal of the coefficient of the second term is

$$\omega\tau_C = \frac{1}{\omega\tau_L} = \sqrt{\frac{\tau_C}{\tau_L}} = \frac{R}{X} = \Gamma_{\text{parallel}}. \quad (14)$$

Here the quality factor is $\Gamma_{\text{parallel}} = R/X$. In figure 2, the current diverted through the resistor will be small (so that the resistor only weakly dampens the oscillations of the LC circuit) if $R \gg X$, again corresponding to a high quality factor.

There is another way to rewrite the differential equations, such that their coefficients depend only on the products and ratios of the time constants. Equation (11) is equivalent to

$$\sqrt{\tau_L\tau_C}\ddot{Q} + \sqrt{\frac{\tau_C}{\tau_L}}\dot{Q} + \frac{Q}{\sqrt{\tau_L\tau_C}} = 0 \quad (15)$$

and equation (13) to

$$\sqrt{\tau_L\tau_C}\ddot{Q} + \sqrt{\frac{\tau_L}{\tau_C}}\dot{Q} + \frac{Q}{\sqrt{\tau_L\tau_C}} = 0. \quad (16)$$

These forms of the equations emphasize that τ_L appears just as often as τ_C in the coefficients; it is *not* the case that the series circuit favors τ_L nor that the parallel circuit favors τ_C .

The solution of the differential equation describing the underdamped oscillations of the charge on the capacitor in either circuit is

$$Q(t) = Ae^{-t/2\tau} \cos(\omega't - \phi) \quad (17)$$

where A and ϕ are two integration constants, $\tau = \tau_L$ for the series circuit described by equation (5), and $\tau = \tau_C$ for the parallel circuit described by equation (10). The time constant decreases the angular frequency of the oscillations from ω to ω' given by

$$\omega' = \sqrt{\omega^2 - (2\tau)^{-2}}. \quad (18)$$

Students could be asked to check that equation (17) with (18) is a solution of (5) or (10) by substituting it into the differential equation and performing the required time derivatives. In the case of the series circuit, ω' is real and nonzero if and only if

$$\omega > \frac{1}{2\tau_L} \quad \Rightarrow \quad R < 2X \quad \Rightarrow \quad \Gamma_{\text{series}} > 0.5, \quad (19)$$

² Paulson and Ray [12] also discuss series and parallel RLC circuits in terms of the quality factor, but they define two quantities differently: their time constant is 2τ in the present paper, and they use the damped angular frequency ω' from equation (18) instead of the undamped ω in their definitions of Γ .

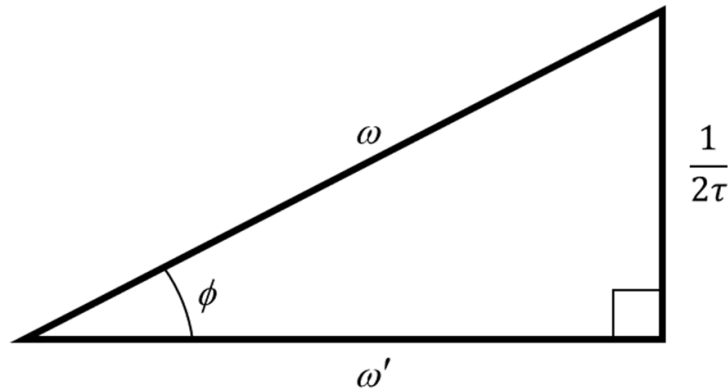


Figure 3. A helpful triangle diagram summarizing the relationship between the undamped angular frequency ω , the damped angular frequency ω' , the decay time constant τ , and the phase constant ϕ of the charge oscillations.

consistent with the discussion in connection with equation (12) that a sufficiently large quality factor is required to achieve weak damping. (One can say that a circuit has *high quality* if it oscillates for *many cycles* before its signal diminishes to negligible levels.) Similarly, for the parallel circuit, ω' is real and nonzero if and only if

$$\omega > \frac{1}{2\tau_C} \Rightarrow R > X/2 \Rightarrow \Gamma_{\text{parallel}} > 0.5, \quad (20)$$

again consistent with the discussion in connection with equation (14) that a sufficiently large quality factor is required for weak damping.

The two constants A and ϕ are determined by the initial conditions. One way to set those conditions is to unwire the capacitor from the circuit temporarily, connect it to a dc power supply of voltage ε , and then replace it in the circuit at a time defined as $t = 0$ with zero initial current out of the capacitor. In other words, $Q(0) = C\varepsilon \equiv Q_0$ and $\dot{Q}(0) = 0$. One might guess that these initial conditions imply $A = Q_0$ and $\phi = 0$ but that is incorrect, because they do not satisfy $\dot{Q}(0) = 0$. Instead, the correct values are³

$$A = Q_0 \frac{\omega}{\omega'} \text{ and } \phi = \cos^{-1} \left(\frac{\omega'}{\omega} \right) \quad (21)$$

as one might instructively ask students to verify. The equation for ϕ can be visualized using the right triangle in figure 3, which is also consistent with equation (18).

3. Experiment

A straightforward way to assemble a series *RLC* circuit is to connect a capacitor (of capacitance C) across a coil of wire. Such a coil will have both inductance L and resistance $R = R_s$ (where the subscript 's' indicates this resistance is in *series* with the inductance). To avoid frequency-dependent losses due to hysteresis (and due to eddy currents to a lesser extent) if the core of the coil were a laminated ferromagnetic bar, it is best to use an air-core coil [8, 9]. A single-pole double-throw switch⁴ is used to connect the capacitor either to a DC power supply of emf ε or to the coil, as sketched in figure 4. To instead make a *parallel RLC* circuit, this circuit also includes a resistor R_p which can be adjusted to any desired value using a variable resistor box. A series *RLC* circuit corresponds mathematically to letting $R_p \rightarrow \infty$ (which in practice is implemented by simply removing the variable resistor box from the circuit). On the other hand, a parallel *RLC* circuit corresponds to letting $R_s \rightarrow 0$ (but which in practice cannot be implemented because the coil has an intrinsic resistance R_s that cannot be eliminated). An oscilloscope probe is connected between points A and B in figure 4, so that it measures the voltage V_C across the capacitor⁵. The oscilloscope is triggered for a single sweep on the descending edge of the signal at a level

³ In addition to setting the initial conditions, ϕ also determines the phase difference between the current to and charge on the capacitor in an underdamped series *RLC* circuit [15].

⁴ Atkin [16] emphasizes the importance of avoiding 'contact bounce' when the switch is thrown from one position to another. This issue is less important on the longer time scales of figures 5 and 6 in this article compared to Atkin figure 2.

⁵ Faleski [17] investigates theoretically the voltages across just the series resistor R_s or just the inductor L , but experimentally those voltages cannot be separately measured because both R_s and L are intrinsic to the coil.

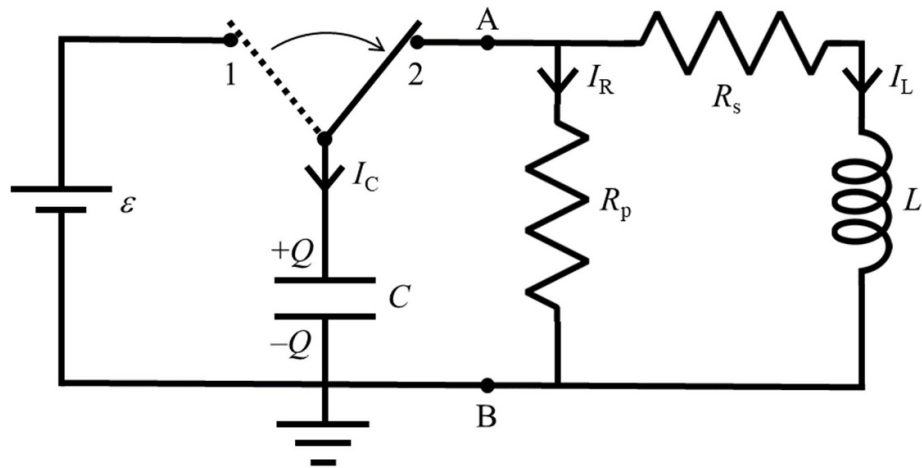


Figure 4. A circuit consisting of a resistor R_s in series with an inductor L , and a resistor R_p in parallel with a capacitor C . A switch is used to initially charge the capacitor up to the emf ε of a DC power supply, and it is then flipped from position 1 to position 2 at time $t = 0$. Subsequently, the time-dependent charges on the capacitor plates are $\pm Q$ and the currents I in the three branches are subscripted in the same way that they are in figure 2. An oscilloscope probe is connected between points A and B to measure the voltage across the capacitor. The power supply, capacitor, probe, parallel resistor, and inductor all share a common connection to ground.

Table 1. Major pieces of equipment used in the experiments. A photograph of these models of multimeter and coil are in figure 1 of Mungan [10].

Item	Model number
Multimeter	Protek 506
Oscilloscope	Agilent DSO-X 2002A with 10:1 voltage probe of 10 M Ω input impedance
Coil	Heath 40–694 of resistance R_s and inductance L with 3400 turns
Capacitor	West-Cap A94W1474-3 of capacitance C rated at 100 V DC
DC power supply	Tenma 72–6615 adjusted to output voltage ε
Variable resistor box	Elenco RS-500 adjusted to resistance R_p

Table 2. Experimental circuit parameters measured using the multimeter for the equipment listed in table 1. Uncertainties are in accordance with the Protek manual (where the errors for the inductances and capacitances are corrected by dividing by the square root of the number of meters used to remeasure their values).

Parameter	Value
L	0.79 ± 0.02 H
C	0.50 ± 0.02 μ F
ε	15.0 ± 0.1 V
R_s	63.0 ± 0.5 Ω
R_p	25.0 ± 0.2 k Ω

slightly below ε (such that the scope records a single trace immediately after the switch is flipped from the charging position 1 to the oscillating position 2). Table 1 summarizes the major items used to build and characterize the circuit in figure 4, and table 2 lists the circuit parameters used in the experiments. (The values of all these parameters were found to be identical to at least two significant figures when measured using several different multimeters except for the inductance and capacitance. Accordingly, the values of L and C reported in table 2 have been averaged over the measurements from more than one multimeter to improve their accuracy.)

For the series circuit, the experimental data is plotted in blue in figure 5. On top of it is plotted the theoretical prediction in red for $V_C = Q/C$ computed from equations (17), (18), and (21) using

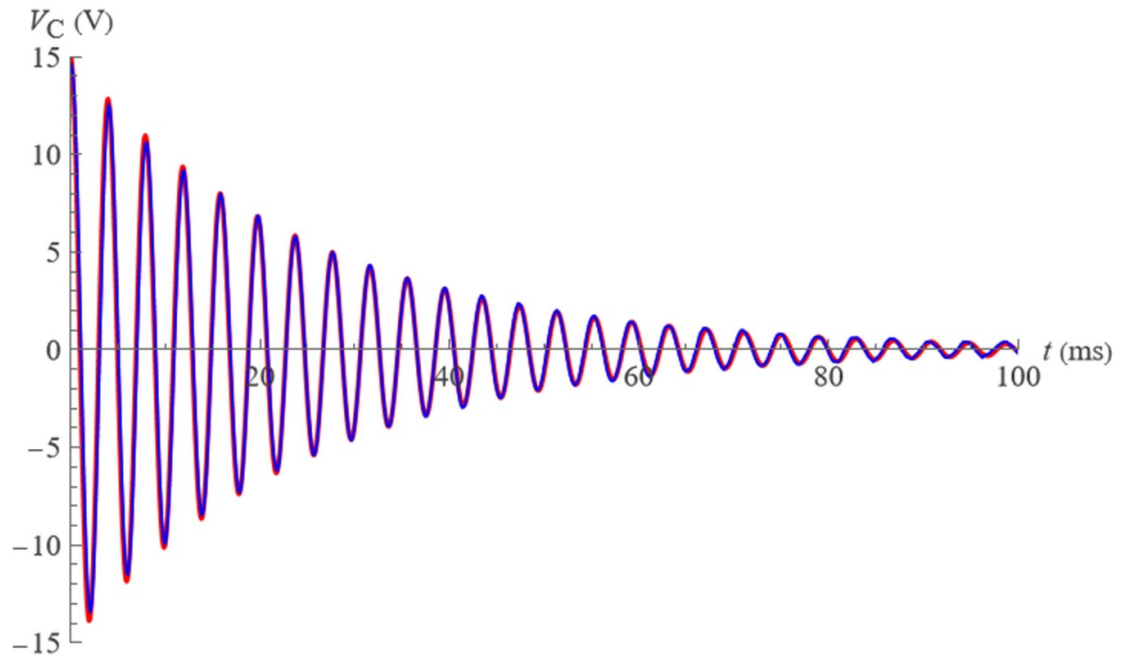


Figure 5. Comparison of the predicted voltage (in red) across the capacitor in figure 1 and the measured voltage (in blue) for the components and parameters listed in tables 1 and 2, where R is the resistance R_s of the coil, and the capacitor was initially charged up to voltage ε using the DC power supply.

$\tau = L/R_s$ from (3), $\omega = (LC)^{-1/2}$ from (1), and $Q_0 = C\varepsilon$ along with the values listed in table 2. To quantify the agreement between experiment and theory, Mathematica is used to perform a nonlinear least-squares fit of these equations to the measured data. That requires only three independent parameters (say $\tau_C = R_s C$, $\tau_L = L/R_s$, and ε) because all other quantities can be expressed in terms of them. The fitting routine returns a high coefficient of determination of 98% with values of $\tau_C = 31.9 \pm 0.2 \mu\text{s}$, $\tau_L = 12.4 \pm 0.1 \text{ ms}$, and $\varepsilon = 14.9 \pm 0.1 \text{ V}$ which agree to within the error bars with $\tau_C = 31.5 \pm 1.5 \mu\text{s}$, $\tau_L = 12.5 \pm 0.4 \text{ ms}$, and $\varepsilon = 15.0 \pm 0.1 \text{ V}$ computed using the values in table 2. The damped period of oscillation is $2\pi/\omega' = 4 \text{ ms}$ and the time for the amplitude to decay from 15 V to $\varepsilon/e = 5.5 \text{ V}$ is $2\tau = 25 \text{ ms}$.

A parallel RLC circuit would have exactly the same signal $V_C(t)$ if the series resistance could be set to zero and a parallel resistance of $R_p = 25 \text{ k}\Omega$ were inserted at the location shown in figure 4. That resistance value comes from equating $\tau_L = L/R_s$ to $\tau_C = R_p C$ or equivalently, equating the geometric average of R_s and R_p to the characteristic impedance $X = (L/C)^{1/2}$ of the oscillator. However, the series resistance cannot be experimentally zeroed out. In lieu of that, the ‘parallel-series circuit’ sketched in figure 4 is constructed having both $R_s = 63 \Omega$ (for the coil) and $R_p = 25 \text{ k}\Omega$ (for the variable resistor box). Given that the series and parallel circuits are each separately described by equation (17), it is not surprising that the combined parallel-series circuit is also, as is proven in the appendix. The theoretical prediction from equation (A8) is plotted in red in figure 6 using the parameters listed in table 2, and the experimental voltage measured using the oscilloscope probe connected between points A and B in figure 4 is plotted on top of that prediction in blue. Performing a nonlinear least-squares fit of the theoretical expressions to the measured data again requires only three independent parameters: say τ_{ps} from equation (A5), ω_{ps} from (A6), and ε . The fitting routine returns a high coefficient of determination of 99.7% with values of $\tau_{ps} = 6.08 \pm 0.01 \text{ ms}$, $\omega_{ps} = 1593.8 \pm 0.1 \text{ rad/s}$, and $\varepsilon = 14.69 \pm 0.02 \text{ V}$ which agree to within the error bars with $\tau_{ps} = 6.26 \pm 0.25 \text{ ms}$, $\omega_{ps} = 1593 \pm 52 \text{ rad/s}$, and $\varepsilon = 15.00 \pm 0.10 \text{ V}$ calculated using the values in table 2, with the exception of ε . The fitted value of the emf is a little less than that directly measured across the power supply because the oscilloscope trigger is set slightly below its value. (The error bars for the fit values include only the statistical fitting uncertainty and do not account for systematic experimental deviations such as that due to the trigger level.) Luke [11] executed similar experiments on a parallel-series RLC circuit, but was limited by the equipment available in 1975 to measurements only of the extrema of the underdamped oscillations for smaller values of L and C than those used here.

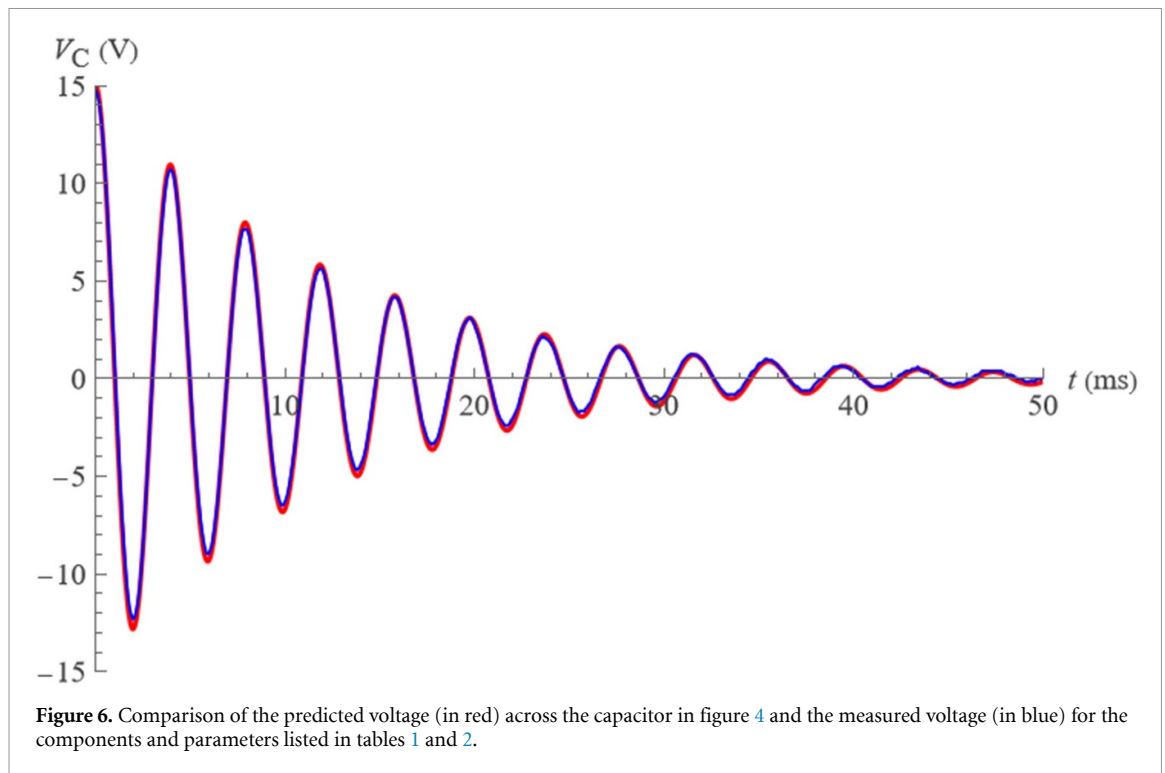


Table 3. Expressions for ω and τ to use in equations (17), (18), and (21) in order to determine the capacitor charge Q (and hence the capacitor voltage $V_C = Q/C$) for the given circuit.

Circuit	Angular frequency	Time constant
Series in figure 1	$\omega = \frac{1}{\sqrt{LC}}$	$\tau = \frac{L}{R}$
Parallel in figure 2	$\omega = \frac{1}{\sqrt{LC}}$	$\tau = RC$
Parallel-series in figure 4	$\omega = \sqrt{\frac{1+R_s/R_p}{LC}}$	$\tau = \left(\frac{1}{R_p C} + \frac{R_s}{L} \right)^{-1}$

4. Conclusion

Predicting the theoretical behavior and measuring the experimental data for a parallel-series RLC circuit are technically appropriate in a standard introductory course that has introduced Kirchhoff's laws and the undriven series RLC circuit. Such investigations can strengthen student understanding of series and parallel circuits in general and of capacitors, inductors, and resistors in particular. Important features of damped oscillations such as the decay time constant, angular frequency, phase constant, and quality factor can be readily calculated and measured using an RLC circuit. These concepts are reinforced by exploring several similar circuits, rather than merely the series version alone.

Specifically, in addition to verifying the equations in this paper, students could be asked to increase R in figure 1 and decrease R in figure 2 to theoretically investigate over and critical (instead of under) damping. They could then increase R_s and/or decrease R_p in figure 4 to experimentally confirm their results.

Several papers [12–14] have considered some of the limitations of idealized circuit models such as that of figure 4. The following issues are particularly relevant to the present article. The capacitor, switch, and wiring between components add a couple of ohms of parasitic resistance, but that is small in comparison to R_s and R_p . Given that the agreement between the measured data and the theoretical predictions in the Experiment section is mostly well within the error bars, it is not necessary to increase the uncertainties to account for that extra bit of resistance. Likewise, the oscilloscope probe has an input impedance of $10\text{ M}\Omega$ which is sufficiently larger than $R_p = 25\text{ k}\Omega$ that the probe does not drag down the measured values of the voltage across the capacitor.

This paper has analyzed the underdamped oscillations of three different kinds of RLC circuits. For the convenience of readers, table 3 summarizes the two fundamental parameters that vary between these three circuits, expressed in terms of the resistances, inductance, and capacitance.

Acknowledgment

Thanks to John Becker for pointing out that many electrical engineering circuits textbooks (in contrast to physics textbooks) analyze the circuit in figure 2. The views expressed in this article are those of the author and do not reflect the official policy or position of the US Naval Academy, the Department of the Navy, the Department of War, or the US Government.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix

Consider the circuit sketched in figure 4. The switch has been in position 1 for a long time, so that the capacitor has charged up to $Q(0) = C\varepsilon \equiv Q_0$, and the switch is then flipped to position 2 at a time defined to be $t = 0$. The capacitor branch current is zero at that instant, $I_C(0) = 0$. At later times, as in the case of figure 2, the direction of I_C gives a positive time derivative, $I_C = +\dot{Q}$, for the charge on the capacitor. Analyze the circuit when the switch is in position 2 using Kirchhoff's rules as follows.

The (upward) voltage across the capacitor equals that across the parallel resistor, so that

$$\frac{Q}{C} = I_R R_p \quad \Rightarrow \quad I_R = \frac{Q}{R_p C}. \quad (\text{A1})$$

The sum of the three downward currents is zero,

$$I_C + I_R + I_L = 0 \quad \Rightarrow \quad I_L = -\frac{Q}{R_p C} - \dot{Q}. \quad (\text{A2})$$

using equation (A1) and $I_C = \dot{Q}$. The voltage across the capacitor equals that across the coil (having resistance R_s and inductance L),

$$\frac{Q}{C} = I_L R_s + L\dot{I}_L \quad \Rightarrow \quad \frac{Q}{C} = -\frac{R_s}{R_p C} Q - R_s \dot{Q} - \frac{L}{R_p C} \dot{Q} - L\ddot{Q} \quad (\text{A3})$$

using equation (A2). Equation (A3) rearranges into the same form as (5) and (10), namely

$$\ddot{Q} + \frac{\dot{Q}}{\tau_{ps}} + \omega_{ps}^2 Q = 0 \quad (\text{A4})$$

where the 'parallel-series' time constant is

$$\tau_{ps} = \left(\frac{1}{R_p C} + \frac{R_s}{L} \right)^{-1} \quad (\text{A5})$$

and the undamped angular frequency is

$$\omega_{ps} = \sqrt{\frac{1 + R_s/R_p}{LC}}. \quad (\text{A6})$$

As a check, τ_{ps} reduces to τ_L in equation (3) with $R = R_s$, and ω_{ps} reduces to ω in equation (1), if $R_p \rightarrow \infty$ such that figure 4 becomes the series circuit of figure 1. Likewise, τ_{ps} reduces to τ_C in equation (2) with $R = R_p$, and ω_{ps} reduces to ω in equation (1), if $R_s = 0$ such that figure 4 becomes the parallel circuit of figure 2.

Since the differential equation for $Q(t)$ has the same form as found previously, it has the same solution given by equation (17) where $\tau = \tau_{ps}$, ω' is given by (18) with $\omega = \omega_{ps}$, and A and ϕ are given by (21). This expression for ω' simplifies to

$$\omega' = \sqrt{\frac{1}{LC} - \frac{1}{4} \left(\frac{1}{R_p C} - \frac{R_s}{L} \right)^2}. \quad (\text{A7})$$

For the special case of the parameters in table 2, where R_p was chosen so that $R_p C = L/R_s$, the solution for the capacitor voltage Q/C further simplifies to

$$V_C = \varepsilon \left(1 + R_s/R_p\right)^{1/2} e^{-tR_s/L} \cos \left[(LC)^{-1/2} t - \cos^{-1} \left(1 + R_s/R_p\right)^{-1/2} \right]. \quad (\text{A8})$$

ORCID iD

Carl E Mungan  [0000-0001-7084-5402](https://orcid.org/0000-0001-7084-5402)

References

- [1] Tipler P A and Mosca G 2020 *Physics for Scientists and Engineers* 6th edn (Macmillan)
- [2] Hudson A and Nelson R 1990 *University Physics* 2nd edn (Saunders) sec 34.5
- [3] Marion J B and Hornyak W F 1982 *Physics for Science and Engineering* (Saunders) sec 37.5
- [4] Alexander C K and Sadiku M N O 2013 *Fundamentals of Electric Circuits* 6th edn (McGraw Hill) ch 8
- [5] Ulaby F T and Maharbiz M M 2010 *Circuits* (NTS Press) ch 6
- [6] Nilsson J W and Riedel S A 2000 *Electric Circuits* 6th edn (Prentice Hall) ch 8
- [7] Mayergoyz I D and Lawson W 1997 *Basic Electric Circuit Theory* (Academic) ch 7
- [8] Kraftmakher Y 2011 Demonstrations with an LCR circuit *Phys. Teach.* **49** 168–70
- [9] Mungan C E 2026 Flattening the oscillations of an underdamped RLC circuit using a diode *Eur. J. Phys.* **47** 025207
- [10] Mungan C E 2022 Simple but accurate driven RLC experiment *Phys. Educ.* **57** 053002
- [11] Luke K L 1975 Laboratory investigation of free and driven oscillations using a RLC circuit *Am. J. Phys.* **43** 610–4
- [12] Paulson J G and Ray M W 2022 Exploration of the Q factor for a parallel RLC circuit *Am. J. Phys.* **90** 903–7
- [13] Mak S-Y 1994 The RLC circuit and the determination of inductance *Phys. Educ.* **29** 94–97
- [14] Ivković S S, Marković M Z, Ivković D Ž and Cvetanović N 2017 LCR circuit: new simple methods for measuring the equivalent series resistance of a capacitor and inductance of a coil *Eur. J. Phys.* **38** 055705
- [15] Ramachandran V 1991 Revisiting the LCR circuit *Phys. Educ.* **26** 318–21
- [16] Atkin K 2023 LCR transients in theory and practice *Phys. Educ.* **58** 035020
- [17] Faleski M C 2006 Transient behavior of the driven RLC circuit *Am. J. Phys.* **74** 429–37