

## What is the buoyant force on a block at the bottom of a beaker of water?

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### Abstract

*I propose that buoyant force be generally defined as the negative of the total weight of the fluids that are displaced, rather than as the net force exerted by fluid pressures on the surface of an object. In the case of a body fully surrounded by fluids, these two definitions are equivalent. However, if the object makes contact with a solid surface (such as the bottom of a beaker of liquid), only the first, volumetric definition is well defined while the second definition ambiguously depends on how much fluid penetrates between the object and the solid surface.*

Several recent papers [1-3] have revived questions about the nature of the buoyant force on a submerged object that is not fully surrounded by fluid. Suppose it makes contact with a solid surface, such as a rectangular block firmly pressed to the bottom of a beaker of water. An earlier pair of papers [4-5] suggests that in such a case the buoyant force has been *removed*. Others argue that while a buoyant force still exists, its direction is now *downward* [6].

The logic behind both of these viewpoints is evident, but which one is consistent with introductory physics textbooks? Open your favorite text and see if it answers this question. You will probably find that it does not. Conventional books introduce buoyant force by considering an object suspended in a liquid (perhaps by a string of negligible cross-sectional area) so that it is fully surrounded by a single fluid.

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Alternatively the body is floating and is thus surrounded by two fluids. But the question of what happens when an object is only partly surrounded by fluid is passed over in silence.

This silence leads to a nontrivial pedagogical issue for introductory physics [7]. Consider drawing a free-body diagram for a block on a table including the effects of the atmosphere. Should this diagram include a buoyant force and, if so, in what direction [8]?

To resolve this ambiguity, we need to *clarify the definition* of buoyant force [9]. Consider the following model situation. A block of lower density than a fluid is held down at the bottom of a beaker of the fluid as a result of the reduced pressure inside a suction cup [10] (or thin o-ring) spanning the block's bottom face. The block has mass  $m$  and top and bottom surfaces of area  $A$ , while the plastic of the suction cup has negligible mass and volume. Define  $P_{\text{top}}$  to be the fluid pressure at the depth of the top surface of the block, and let  $P_{\text{bottom,inside}}$  and  $P_{\text{bottom,outside}}$  be the fluid pressures at the depth of the bottom surface of the block respectively inside and outside the volume of fluid enclosed by the suction cup. The existence of suction implies that

$$P_{\text{bottom,inside}} < P_{\text{bottom,outside}}.$$

The weight of the block is balanced by the difference in fluid forces on the bottom and top of the block and by a normal force  $N$  exerted by the semi-rigid side walls of the suction cup,

$$mg = N + AP_{\text{bottom,inside}} - AP_{\text{top}}. \quad (1)$$

We can express the pressure inside the suction cup as the difference between the pressure in the surrounding fluid at the same depth and the pressure differential across the membrane of the suction cup,

$$P_{\text{bottom,inside}} = P_{\text{bottom,outside}} - \Delta P.$$

This can be substituted into Eq. (1) to obtain

$$mg = N + B - F, \quad (2)$$

where the magnitude of the buoyant force  $B$  has here been defined to be equal to the weight of fluid displaced by the block, and  $F = A\Delta P$  is the “holding” force due to the suction.

Although Eqs. (1) and (2) are fully equivalent and both contain exactly one upward and one downward fluid force term, there are three advantages of the second equation over the first:

1. We can use Eq. (2) to immediately compute the minimum force  $F_{\text{min}}$  required to hold the block down, by setting  $N = 0$ . One finds the intuitively appealing result that it is equal to the negative of the block's apparent weight  $mg - B$ . In contrast, the hold-down force is not explicit in Eq. (1).
2. We have separated the variation in fluid pressure with depth from the pressure differential  $\Delta P$  due to the suction. This is a pedagogically instructive distinction to make.
3. Equation (2) consistently *defines* the buoyant force on an immersed object to be upward and equal in magnitude to the weight of fluid displaced, even when the object makes contact with solid surfaces [11]. This definition remains simple and unambiguous if  $\Delta P$  is non-zero.

Equation (2) also holds for a block (of arbitrary density) on a table, if we broaden  $F$  to include not just the force resulting from suction, but also from such effects as surface tension, cold welding, and electrostatic surface charge interactions when appropriate [12]. But usually these effects are negligible. With that understanding, it is reasonable to ask students, “What is the magnitude of the force required to slowly lift the block?” A good first approximation is its weight  $mg$ . If the problem asks us to account for the effects of the fluid environment (such as the atmosphere) on an ordinary block, the correct answer would then be its apparent weight  $mg - B$  [13].

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It is only when there is a reasonable expectation that the block is somehow coupled or sealed [14] to the table that one needs to include additional forces  $F$ . This is entirely analogous to how projectiles are treated in introductory physics: One initially takes them to be in freefall. Subsequently a velocity-dependent drag force can be added to account for air resistance. But additional effects such as lift are only modeled under special circumstances.

### References

- [1] J. Bierman and E. Kincanon, "Reconsidering Archimedes' principle," *Phys. Teach.* **41**, 340-344 (2003).
- [2] B.M. Valiyov and V.D. Yegorenkov, "Do fluids always push up objects immersed in them?" *Phys. Educ.* **35**, 284-286 (2000).
- [3] E.H. Graf, "Just what did Archimedes say about buoyancy?" *Phys. Teach.* **42**, 296-299 (2004).
- [4] G.E. Jones and W.P. Gordon, "Removing the buoyant force," *Phys. Teach.* **17**, 59-60 (1979).
- [5] J.R. Ray and E. Johnson, "Removing the buoyant force: A follow-up," *Phys. Teach.* **17**, 392-393 (1979).
- [6] "Downward" and "upward" are relative to the direction of the effective gravitational field  $\mathbf{g}$  including the acceleration of the reference frame. On a carousel, for example, "upward" is tilted toward the axis of rotation, as shown in H.J. Haden, "A demonstration of Newtonian and Archimedean forces," *Phys. Teach.* **2**, 176-177 (1964). As another example, on an elevator accelerating downward, the weight of and buoyant force on a submerged object are decreased.
- [7] J. Harper, "Archimedes' principle and the FCI," *Phys. Teach.* **41**, 510 (2003).
- [8] Place a block of mass  $m$  on the submerged tray of a spring scale, as described in P.A. Tipler and G. Mosca, *Physics for Scientists and Engineers*, 5th ed. (New York, Freeman, 2003), Sec. 13-3. Tare the scale while the block is submerged but before it contacts the tray. If you now place the block on the tray, the scale reading will be  $mg - B$  where  $B$  is the weight of fluid displaced by the block. This neither proves nor disproves that there is a buoyant force on the block *alone*. For example, if fluid is squeezed out between the block and tray, the downward fluid force on the block *increases* and that on the tray *decreases* by exactly the same amount. That is, the scale is actually sensitive to the buoyant force on the *combination* of the block and tray, but its reading results from the manner of taring. (For further discussion of methods of weighing a block in contact with the bottom of a fluid, see Ref. 3.)
- [9] The integral of the fluid pressure over the surface of a body could be sideways (e.g., on a submerged block in contact with the wall of an aquarium). Therefore taking that to be the general definition of buoyant force would be misleading: A dictionary defines "buoyant" to mean upward, as the reader is invited to check. In any case such an integral cannot be evaluated if the extent of fluid seepage at the solid interface is unknown.
- [10] Interestingly, the adhesion of many common tapes results from suction, as discussed in R. Kunzig, "The physics of tape," *Discover* **20**, 27-29 (July 1999).
- [11] Archimedes' principle is deduced in R.E. Vermillion, "Derivations of Archimedes' principle," *Am. J. Phys.* **59**, 761-762 (1991) by considering the net change in the gravitational potential energy of *both* the fluid and block during a virtual upward displacement of the block. No fluid need initially be under the block for this derivation to hold.
- [12] For example, if fluid seepage under the block can occur, so that suction cannot be sustained, then some other effect such as a glue must exert force  $F$  to hold down a block whose density is less than that of the fluid. This is consistent with my analysis in C.E. Mungan, "Reprise of a 'Dense and tense story'," *Phys. Teach.* **42**, 292-294 (2004).
- [13] The effect of atmospheric buoyancy on scale readings is memorably illustrated in H. Stokes and W.D. Peterson, "Buoyancy of air: An attention-grabbing demonstration," *Announcer* **33**, 94 (Summer 2003), as described on the web at <http://stokes.byu.edu/alkaseltzer.html>.
- [14] A striking demonstration of an object sealing to a surface is the "Atmospheric Pressure Demonstrator" currently sold by PASCO. It consists of a rubber sheet (with a knob attached to its top) that when slapped down onto a stool can be used to lift it.

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