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Comment on 'Work done by sliding friction on an incline'

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Abstract
When an object slides across a surface, macroscopic mechanics cannot predict how the total thermal energy that is generated gets partitioned between the object and the surface. However, thermodynamics defines work as a quantity of energy gained (or lost) by one system with an equal amount lost (or gained) by another system (called the surroundings of the first system) as a result of a force acting between them. Using this definition, the work that the object (taken to be the system) performs on the surface (taken to be the surroundings) due to sliding friction cannot be determined solely from knowledge of the initial and final bulk mechanical energies involved.

Keywords: work, mechanical energy, thermodynamics, systems

In a recent paper [1], Rod Cross analyzes the energetics of a ball rolling while slipping down an incline. The symbolic calculations are clearly explained, sample numerical results are helpfully graphed, and the article is attractively brief.

Despite these features, it can be confusing to present the material in a physics course in the way Cross does. He equates calculations of 'work' to changes in the translational and rotational kinetic energies of the ball. This approach is not in harmony with the presentation of the topic of work and energy in standard introductory physics textbooks because of the dissipative nature of sliding friction. His approach can lead to difficulties when dealing with deformable objects, fluids, fields, and other such systems that are not particles or rotating rigid bodies.

Cross correctly determines the relationship between 'mechanical variables' such as time, position*, velocity*, acceleration*, force*, mass*, momentum*, kinetic energy*, potential energy, and mechanical energy. (Quantities marked with an asterisk include both their linear forms and their angular analogs). However, the interpretation of his results within the framework of thermodynamics is unsatisfying. Introductory textbooks often qualify the way they introduce 'work' in the mechanics chapters (such as by restricting attention to particles) with a view toward its later appearance in thermodynamics, where it is defined as a means of transferring energy *across the boundary between systems*.

The view of 'work' adopted by Cross is helpful as a tool for solving Newton's second law. It serves as a reminder that it is useful to integrate (which in Cross's problem simply means to multiply) a force (or torque) over a

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relevant displacement. This strategy is successful, for instance, in treating a spring pushing on a block that is moving on an air table (where the system of interest is the block). The work done by the spring equals the change in kinetic energy of the block. Unfortunately, the same logic fails when applied to the slow compression of a gas in a cylinder (where the system of interest is the gas). The gas does not gain mechanical energy, and yet work was clearly done to compress it.

In the case of a ball rolling with slipping down a ramp, Cross refers to the 'work done by friction' which is a phrase that many modern textbooks eschew because it does not equate to a quantity of energy transferred between the ramp and the ball.

Cross appears to believe he has evaded the issue of multiple points of frictional contact by postulating a perfectly spherical ball rolling down a perfectly flat ramp. However, the contact pressure will flatten the ball and indent the ramp slightly. Even if that did not happen at low temperatures, their atoms will start vibrating as the ball and ramp warm up and so they will not remain perfectly spherical and flat.

His solution can be summarized as follows, with reference to his free-body diagram. The linear acceleration a of the ball down the incline (making angle θ with the horizontal) is the difference between the component of gravity parallel to the ramp and kinetic friction up the ramp, so that $a = (\sin\theta - \mu\cos\theta)g$ where μ is the coefficient of kinetic friction. This constant a determines the speed v of the ball (of mass m) after it descends a distance s along the incline starting from rest via the kinematic relation $v^2 = 2as$, resulting in the translational kinetic energy $K_\text{T} = mg(\sin\theta - \mu\cos\theta)s$. In analogous manner, the angular acceleration α equals the torque (about the center of mass) $\mu mgR\cos\theta$ divided by the moment of inertia kmR^2 (where $k = 0.4$ for a solid ball). The angular speed ω after the ball rotates through an angle ϕ is $\omega^2 = 2\alpha\phi$, giving the rotational kinetic energy as $K_\text{R} = (\mu mgR\cos\theta)\phi$. This result can be rewritten in terms of s using the ratio $\phi/s = \alpha/a = \mu(kR)^{-1}(\tan\theta - \mu)^{-1}$. Finally, the thermal energy generated equals the mechanical energy lost, given by $mgs\sin\theta - (K_\text{T} + K_\text{R})$ because the first term on the right is the loss in gravitational potential energy. There is no need to muddy this elegant analysis by invoking 'work'. Notably, none of these calculated quantities can be used to determine what portion of the generated thermal energy ends up in the ball and what portion in the ramp. Simplifying the problem to an isolated interaction between a block sliding across a horizontal table (thereby eliminating potential energy and rotational kinetic energy from consideration), it is then even clearer that the concept of work is not helpful in determining the final states of the block and table [2].

The bottom line is there are two forms of work of interest in introductory physics: one associated with changes in mechanical energy of an object (such as in the work-kinetic-energy theorem and in the definition of potential energy), and another associated with transfers of energy from one system to another. While in some problems these two forms of work are equivalent, in general they are not and so both cannot be called 'work' without qualification. Textbooks often give priority to the definition of work which appears in the first law of thermodynamics, rather than to computational quantities (helpful though they are) that arise from integrating Newton's second law.

The views expressed in this Comment are those of the author and do not reflect the official policy or position of the US Naval Academy, the Department of the Navy, the Department of War, or the US Government.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Work done by sliding friction on an incline

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Abstract

When a ball slides down an incline, the friction force acts to dissipate kinetic energy. The total mechanical work done by the friction force is equal to the decrease in kinetic energy, and is determined by the displacement of the point of application of the friction force. The friction force does work to increase the rotational kinetic energy and to decrease the translational kinetic energy of the ball. The change in the translational kinetic energy is determined by the displacement of the centre of mass. Two different work-energy theorems are therefore needed to describe the work done by the friction force.

Keywords: sliding friction, energy dissipation, kinetic energy

1. Introduction

The work-energy theorem presented to high school students usually involves the work done on single particle, in which case the work done by a force F to change its translational kinetic energy is Fs where s is the displacement of the particle. If a force is exerted on a larger object then the object might also rotate, in which case additional work is done by F to change its rotational kinetic energy. Then the total work done by F is Fx where x is the displacement of the point of application of F. Part of the total work involves the displacement of the centre of mass, s, in which case the change in the translational kinetic energy is still Fs and the other part involves a change in the rotational kinetic energy.

It is well known that energy is conserved when a ball rolls without sliding down an incline since the friction force is due to static friction. The displacement of the point of application of the force at the bottom of the ball is zero, so the total work done by the friction force is zero. Work is done by the friction force to increase the rotational kinetic energy of the ball, but it is equal and opposite the work done by the friction force to reduce the translational kinetic energy of the ball [1, 2]. The total kinetic energy at the bottom of the incline is equal to the decrease in potential energy from the top to the bottom of the incline.

At high angles of inclination, the ball slides down the incline, in which case the point of application of the force does not remain at rest and energy is dissipated by sliding friction. It is likely that students would have a problem in determining the work done by the friction force in that situation, given that work, rotational motion and friction are all difficult concepts at an introductory level. As with static friction, two different work-energy theorems are needed to describe the work done by a sliding friction force, one involving the displacement of the centre of mass and the other involving the displacement of the point of application of the force.

2. Equations of motion

The problem can be solved directly, in terms of the equations of motion. The geometry is shown in figure 1. Three forces act on the ball: the gravitational force $mg\sin\theta$ acting down the incline, the sliding friction force, F, acting up the incline, and the normal reaction force $N = mg\cos\theta$ where m is the mass of the ball. The equations of motion are

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = ma = mg\sin\theta - F \quad (1)$$

and

$$I_{\mathrm{cm}}\frac{\mathrm{d}\omega}{\mathrm{d}t} = FR \quad (2)$$

where $F = \mu N$, μ is the coefficient of sliding friction, R is the ball radius, $I_{\mathrm{cm}} = kmR^2$ is the moment of inertia of the ball for rotation about its centre of mass and FR is the friction torque acting on the ball. For a uniform, solid ball, $k = 2/5$. The constant k depends on the mass distribution of the ball and is different for a hollow ball.

Equation (1) describes the change in the translational velocity of the ball and equation (2) describes the change in its rotational velocity. The linear acceleration of the ball is given by

$$a = g(\sin\theta - \mu\cos\theta)\,. \quad (3)$$

If the ball starts from rest then the velocity, v, of the ball after it travels a distance s down the incline is given by $v^2 = 2as$ so its translational kinetic energy is

$$\begin{aligned}\frac{1}{2}mv^2 &= mg(\sin\theta - \mu\cos\theta)s = mgs\sin\theta - Fs \\ &= mgh - Fs\,, \end{aligned} \quad (4)$$

where $h = s\sin\theta$ is the initial height of the ball. Equation (4) indicates that the work done by the gravitational force to increase the translational kinetic energy is $mg\sin\theta s$ and is equal to the decrease in potential energy of the ball, but negative work is done by F to decrease the translational kinetic energy. The work done in each case is given by the force multiplied by the displacement of the centre of mass. After a time, t, the distance s is given by

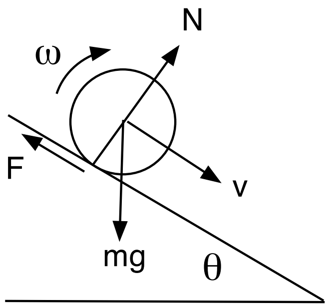

Figure 1. A ball sliding down an incline.

$$s = \frac{1}{2}at^2 = \frac{g}{2}(\sin\theta - \mu\cos\theta)t^2\,, \quad (5)$$

and the angular velocity is given by

$$\omega = \frac{FRt}{I_{\mathrm{cm}}} = \frac{\mu g\cos\theta\, t}{kR}\,. \quad (6)$$

The rotational kinetic energy of the ball is then

$$\frac{1}{2}I_{\mathrm{cm}}\omega^2 = Fs \times \frac{\mu}{k(\tan\theta - \mu)} \quad (7)$$

where t^2 was eliminated using equation (5), together with the relation $F = \mu mg\cos\theta$. The total kinetic energy at the bottom of the incline is the sum of equations (4) and (7), or

$$KE = (mgh - Fs) + Fs \times \frac{\mu}{k(\tan\theta - \mu)}\,. \quad (8)$$

Energy is conserved if $\mu = k(\tan\theta - \mu)$, which marks the transition from rolling without slipping to a slipping phase. If $\tan\theta > (1+k)\mu/k$ then the ball will slip and the loss in energy (or the energy converted to thermal energy) is given by the potential energy at the top of the incline minus the kinetic energy at the bottom of the incline. That is, the energy loss is given by

$$mgh - (mgh - Fs) - Fs \times \frac{\mu}{k(\tan\theta - \mu)} = Fs\left[1 - \frac{\mu}{k(\tan\theta - \mu)}\right]. \quad (9)$$

The total mechanical work done by the friction force is equal to the friction force multiplied by the displacement of its point of application at the contact point on the incline. The contact point at the bottom of the ball travels down the incline at a speed v_P given by

$$v_P = \frac{\mathrm{d}x_P}{\mathrm{d}t} = v - R\omega = g(\sin\theta - \mu\cos\theta)t - \frac{\mu g\cos\theta\, t}{k}, \quad (10)$$

so

$$x_P = \frac{g}{2}\left(\sin\theta - \mu\cos\theta - \frac{\mu\cos\theta}{k}\right)t^2. \quad (11)$$

Elimination of t^2 using equation (5) gives the result that

$$Fx_P = Fs\left[1 - \frac{\mu}{k(\tan\theta - \mu)}\right], \quad (12)$$

which is the same as equation (9) and indicates that the total mechanical work done by F is equal to the energy converted to thermal energy of both the ball and the incline. The quantity Fx_P is sometimes described as the work done by F [3] but is better described as the total mechanical work done by F since work is done by F to change both the translational and rotational kinetic of the ball. Fx_P therefore represents the total work done by F to change the translational and rotational kinetic energy of the ball, and it is equal to the energy converted to heat. By contrast, the work done by F to decrease the translational kinetic energy is Fs and the work done by F to increase the rotational kinetic energy is given by equation (7), which is less than Fs if the ball is sliding but equal to Fs if the ball rolls without sliding. In the latter case, F is a static friction force, $x_P = 0$ and the total mechanical work done by F is zero since the work done by F to increase the rotational kinetic energy is equal and opposite the work done to decrease the translational kinetic energy [1, 2]. Consequently, no thermal energy is generated when a ball rolls without slipping down an incline.

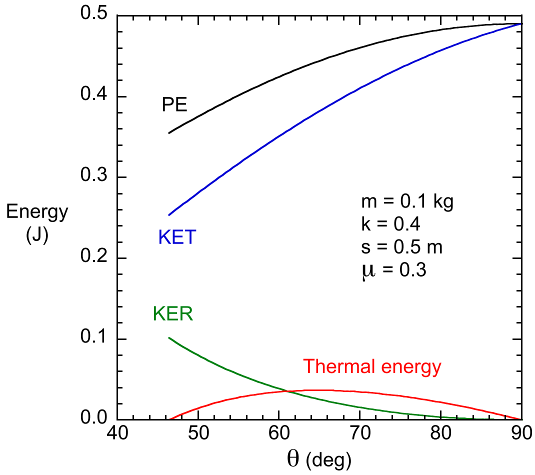

Figure 2. Theoretical results when $m = 0.1$ kg, $s = 0.5$ m and $\mu = 0.3$ where PE $= mgs\sin\theta$ is the potential energy at the top of the incline, KET $= \frac{1}{2}mv^2$ is the translational kinetic energy at the bottom of the incline, KER $= \frac{1}{2}I_{\mathrm{cm}}\omega^2$ is the rotational kinetic energy at the bottom of the incline and Thermal energy $=$ PE $-$ KET $-$ KER $= Fx_P$ is the energy dissipated due to sliding friction.

A theoretical calculation is shown in figure 2 for a $m = 0.1$ kg ball with $k = 2/5$ sliding down an incline of length $s = 0.5$ m with $\mu = 0.3$. In that case, the ball slides if $\theta > 46.4°$. As θ increases, the initial potential energy of the ball increases, and so does the translational kinetic energy at the bottom of the incline, but the rotational kinetic energy at the bottom of the incline decreases. The thermal energy generated by the sliding motion reaches a maximum value and then decreases to zero at $\theta = 90°$ since the sliding friction force then decreases to zero.

The work done by friction on a block sliding down an incline or across a floor is more complicated since there are many contact points between the block and the incline, as described by Sherwood and Bernard [4] and by Poon [5], so the displacement of the point of application of the friction force is not well defined. Nevertheless, if a block of mass M is projected at speed v_0 across a horizontal floor and slides to a stop in a distance x then the decrease in its translational kinetic energy is $Fx = \frac{1}{2}Mv_0^2$ where F is the sliding friction force acting on the block. The mechanical work done by F is $-Fx$ and the thermal energy generated is Fx.

3. Conclusion

The total mechanical work done by the friction force on a ball that slides down an incline includes the work done to change its translational kinetic energy plus the work done to change its rotational kinetic energy. The work done to change its translational kinetic energy can be calculated from the displacement of its centre of mass, while the total mechanical work done by the friction force can be calculated from the displacement of the point of application of the force. The work done by the friction force results in a decrease in the kinetic energy of the ball which appears as thermal energy in the ball and the incline.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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