COMMENT

Phys. Educ. **61** (2026) 038003 (4pp) iopscience.org/ped

Comment on 'Work done during a head-on elastic collision'

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Abstract

A general model for a one-dimensional elastic collision between two balls is proposed, and the relevant definitions and calculations of work, kinetic energy, and potential energy are clarified.

Keywords: elastic collision, work, work-kinetic-energy theorem, potential energy, first law of thermodynamics

In a recent paper [1], Rod Cross analyzes two balls elastically colliding head-on as if they were two rigid spheres of masses m_1 and m_2 interacting via a massless Hookean spring of stiffness constant k. It is instructive to instead treat the balls as two massive springs. Each ball can then be modelled as an infinite set of point particles of infinitesimal mass m_i connected by infinitesimally long, massless Hookean springs of stiffness constant k_i (where $i = 1, 2, 3, \ldots$). That might seem complicated, but it suffices to consider the situation only while the two balls are in contact with each other, resulting in a *single* overall chain of alternating particles and ideal springs. Further, it is enough to treat the case of *three* particles connected by *two* springs, as sketched in figure 1, because the results are easily generalised to any number N of springs connecting $N + 1$ particles together[1].

Following Cross, it is helpful to analyze the dynamics in terms of work and mechanical energy. Define the *work* done by an object 1 as a result of the contact force $\vec{F}_{12}$ it exerts on point C of an object 2 during some interaction (which can generally be considered to be a collision) between the two objects as the line integral,

$$W_{12} = \int \vec{F}_{12} \cdot \mathrm{d}\vec{r} \tag{1}$$

where $\mathrm{d}\vec{r}$ is the differential displacement of the contact point C. By Newton's

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[1] There must be particles and not springs at the two ends of the chain, as in figure 1. If there were a spring at an end it could not change length (since there is no external force on a free end) and it would thus behave like a rigid segment of the spring (and therefore like a particle).

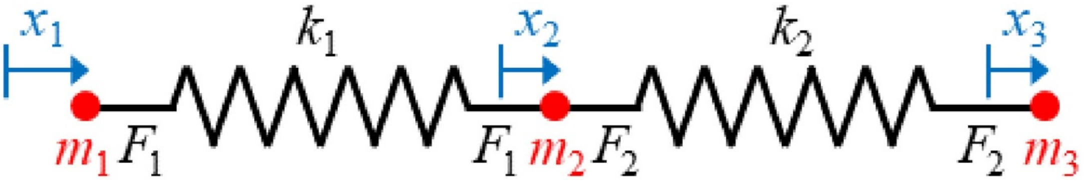

Figure 1. Three point particles of masses m_1, m_2, and m_3 connected by two massless Hookean springs of stiffness constants k_1 and k_2. The initial equilibrium positions of the three particles at the instant of collision are indicated by the three short vertical lines; as a result of the motion of the balls during the collision, the displacements of the particles from those initial positions are x_1, x_2, and x_3. Thus particle 1 has velocity $\upsilon_1 = \mathrm{d}x_1/\mathrm{d}t$ and acceleration $a_1 = \mathrm{d}^2x_1/\mathrm{d}t^2$ (and likewise for particles 2 and 3). Since the springs are massless, the net force on them must be zero (according to Newton's second law) and thus the tension must be constant all along the length of each spring. The tension at the left end of spring 1 is F_1 pushing leftward on m_1 and rightward on k_1, and the tension at the right end of spring 1 is also F_1 pushing leftward on k_1 and rightward on m_2 (and likewise for F_2 at the two ends of spring 2). The positive direction for vector quantities x, υ, a, and F is rightward.

third law, the corresponding work that object 2 does on object 1 is,

$$W_{21} = \int \vec{F}_{21} \cdot \mathrm{d}\vec{r} = -W_{12}. \tag{2}$$

The opposite signs of W_{12} and W_{21} are consistent with the fact that if one object loses energy[2] during the collision, the other object must gain an equal amount of energy.

In particular, consider the work that spring 1 exerts on particle 1 in figure 1,

$$\begin{aligned} -\int F_1 \, \mathrm{d}x_1 &= \int m_1 \frac{\mathrm{d}\upsilon_1}{\mathrm{d}t} \mathrm{d}x_1 \\ &= \int m_1 \upsilon_1 \, \mathrm{d}\upsilon_1 = \frac{1}{2} m_1 \Delta\left(\upsilon_1^2\right) \end{aligned} \tag{3}$$

where the minus sign stems from the fact that F_1 acts leftward on m_1 but the displacement of that particle is rightward. Equation (3) is a statement of the work-kinetic-energy theorem for particle 1 in the form,

$$W_{\text{net on particle 1}} = \Delta K_1 \tag{4}$$

where K_1 is the kinetic energy of particle 1. Likewise consider the sum of the work that springs 1 and 2 exert on particle 2,

$$\begin{aligned} &\int F_1 \, \mathrm{d}x_2 - \int F_2 \, \mathrm{d}x_2 \\ &\quad = \int F_{\text{net on particle 2}} \, \mathrm{d}x_2 \\ &\quad = \int m_2 \frac{\mathrm{d}\upsilon_2}{\mathrm{d}t} \mathrm{d}x_2 = \tfrac{1}{2} m_2 \Delta\left(\upsilon_2^2\right) \end{aligned} \tag{5}$$

which can be summarised as,

$$W_{\text{net on particle 2}} = \Delta K_2. \tag{6}$$

Finally for particle 3 one obtains,

$$\begin{aligned} &\int F_2 \, \mathrm{d}x_3 = \tfrac{1}{2} m_3 \Delta\left(\upsilon_3^2\right) \\ &\qquad \Rightarrow W_{\text{net on particle 3}} = \Delta K_3. \end{aligned} \tag{7}$$

Proceed similarly for the springs. The sum of the work that particles 1 and 2 exert on spring 1 is,

$$\begin{aligned} &\int F_1 \, \mathrm{d}x_1 - \int F_1 \, \mathrm{d}x_2 \\ &\quad = \int F_1 \, \mathrm{d}\left(x_1 - x_2\right) \\ &\quad = \int k_1 \left(x_1 - x_2\right) \mathrm{d}\left(x_1 - x_2\right) \\ &\quad = \frac{1}{2} k_1 \left(x_1 - x_2\right)^2 \end{aligned} \tag{8}$$

which can be rewritten as,

$$W_{\text{net on spring 1}} = \Delta U_1 \tag{9}$$

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[2] This energy loss is not in general the *net* energy change of either object; it is only the energy transfer due to this specific channel of interaction between the two objects.

where U_1 is the potential energy of spring 1. This result is consistent with the definition of the change in elastic potential energy for the conservative spring force, $\Delta U_\text{s} = -W_\text{net by spring}$, given that the net work done on spring 1 by its surroundings is equal and opposite to the net work that spring 1 does on its surroundings[3]. Likewise for spring 2,

$$\int F_2\,\mathrm{d}x_2 - \int F_2\,\mathrm{d}x_3 = \tfrac{1}{2}k_2(x_2 - x_3)^2 \quad \Rightarrow \quad W_\text{net on spring 2} = \Delta U_2. \tag{10}$$

Consistent with what was discussed in connection with equations (1) and (2), the sum of the five preceding net works is zero, because there is zero net external force acting on the system shown in figure 1. As a result, it follows that the sum of the three changes in kinetic energy and the two changes in potential energy equals zero, which is a statement of the conservation of mechanical energy for this isolated system.

To summarise, the net work done on a point particle having mass equals its change in translational kinetic energy, and the net work done on a massless Hookean spring equals its change in elastic potential energy. Energy transferred to an extended rigid body will often additionally involve a change in rotational kinetic energy, and energy transferred to an extended non-rigid body (such as a block sliding across a rough table which agitates asperities on their surfaces, a viscous liquid being stirred, a gas in a cylinder being slowly compressed, or a battery connected to a motor which results in the motion of ions in the electrolyte) often results in changes in non-mechanical energy. The sum of *all* energy transfers and changes for a system is given by the first law of thermodynamics; the work-kinetic-energy theorem and the definition of change in potential energy are special cases of it.

Comparing the preceding analysis to that in Cross’s article, there are two noteworthy differences between them:

1. Cross models the collision of the two balls as two particles of masses m_1 and m_2 connected by one ideal spring of stiffness constant k. In this Comment, the collision is modelled as a large number $N+1$ of particles connected by N ideal springs. In figure 1 here, N was chosen to be 2, because that is already sufficient to illustrate the relevant calculations of work, kinetic energy, and potential energy. However, a realistic model would use a much larger value of N, perhaps 100 or more. There is no limit to how large N can be, and the model presented here becomes exact by letting $N \to \infty$. Importantly, each of these (say) 100 springs will have a different compression during the collision. The compressions will vary both in spatial position (namely, springs inside the balls near the point of collision will be substantially compressed, whereas springs inside ball 1 near its left end or inside ball 2 near its right end will barely be compressed) and in time (namely, the compressions will be small when the balls first make contact with each other and as they separate from each other, whereas they will be a maximum at around the midpoint in time of the collision when the total kinetic energy of the system is a minimum).
2. The concept of work is defined more specifically here than by Cross. Cross states in his abstract that work can be defined *either* as force multiplied by the displacement of the centre of mass of an object, or as force multiplied by the displacement of the point of application

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[3] A similar relationship exists between the work that the gravitational field does on a barbell that is being slowly raised above earth’s surface and the corresponding change in gravitational potential energy [2]. Just as the gain in the elastic potential energy of the spring equals the negative of the net work it does on the two masses at its ends, so the gain in the gravitational potential energy of the gravitational field equals the negative of the net work that gravity does on the two masses at its ends (namely the earth and the barbell, although the work done on the earth is negligibly small because it hardly moves while the barbell is being raised). When masses or charges are moved relative to each other, there is a change in the potential energy stored in the gravitational or electric field existing in the space between and around them.

of the force. Here, work is *only* calculated in the second way[4]. In particular, the first definition cannot be applied to massless springs, because they do not have a 'centre of mass.' Also, Cross's statement at the end of his Introduction that 'the work done by the force on each ball is not the same as the work done by each ball on the spring' could be misconstrued. It depends on whether 'work on the spring' means the *net* work done on the spring by *both* balls, or instead it means the work done on the spring by *one* ball which is how work is calculated in equations (1) and (2) above.

The views expressed in this Comment are those of the author and do not reflect the official policy or position of the US Naval Academy, the Department of the Navy, the Department of War, or the US Government.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Received 26 January 2026, in final form 23 April 2026
Accepted for publication 18 May 2026
https://doi.org/10.1088/1361-6552/ae6f60

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[4] The two definitions are identical for particles. For extended objects, the first definition is called 'pseudowork' or 'centre-of-mass work' in the literature [3] in order to distinguish it from the second definition.

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Phys. Educ. **61** (2026) 023002 (2pp) iopscience.org/ped

Work done during a head-on elastic collision

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Abstract

When two balls collide head-on in an elastic collision, the work done by the impact force can be calculated by modelling the collision as two masses connected by a spring. Work is done on each mass by the spring and work is done on the spring by each mass. The work done in each case depends on whether the work is defined as the force multiplied by the displacement of the centre of mass or whether it is defined as the force multiplied by the displacement of the point of application of the force. Both definitions are valid but give different but complementary results.

Keywords: work-energy theorem, displacements, potential energy

1. Introduction

When two balls collide head-on in a perfectly elastic collision, the outcome can be determined using conservation of momentum and kinetic energy. The work done on each ball was calculated in a previous paper [1] assuming that the collision can be modelled as two rigid masses connected by a spring. The spring was used to simulate the elastic response of the two balls. However, the work done on each ball was calculated by considering only the force exerted by the spring on each ball. That is, the work done by the spring on each ball was calculated, but the work done by each ball on the spring was not examined as a separate issue. The latter calculation is considered in the present paper. Despite the fact that the force exerted by the spring on each ball is equal and opposite the force exerted by each mass on the spring, and the displacements of each ball are the same as the displacements of each end of the spring, the work done by the force on each ball is not the same as the work done by each ball on the spring.

2. Work calculations

The situation is shown in figure 1. Mass m_1 is incident on another mass m_2 initially at rest but a spring separates the two masses. The two balls remain in contact with the spring during the collision but m_1 is shown as being separated from the spring by a small gap so that the force F_2 of m_1 acting on the left end of the spring can be shown more clearly. During the collision, m_1 is displaced by a distance x_1 and m_2 is displaced by a distance x_2.

The left end of the spring exerts a force F_1 on m_1 and $F_2 = -F_1$ so m_1 exerts an equal and opposite force on the left end of the spring. The

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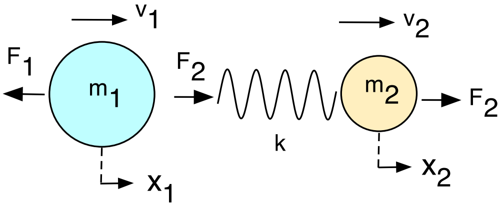

Figure 1. A head-on collision between two balls.

displacement of m_1 is x_1 so the negative work done by F_1 on m_1 is $-\int F_1\,\mathrm{d}x_1$. The displacement of the left end of the spring is also x_1 so $\int F_2\,\mathrm{d}x_1 = -\int F_1\,\mathrm{d}x_1$. However, the work done by F_1 on m_1 is not the same as the work done by F_2 on the spring since the displacement of m_1 refers to the displacement of its centre of mass, whereas the displacement of the spring refers to the displacement of the point of application of F_2. F_2 does work to compress the spring but it does additional work on m_2 to increase its kinetic energy since F_2 is transmitted through the spring to act on m_2 at the right hand end of the spring. That is, the expression $\int F_2\,\mathrm{d}x_1$ refers to the total work done by F_2 to compress the spring plus the work done by F_2 on m_2. The work done on m_2 by the right hand end of the spring is $\int F_2\,\mathrm{d}x_2$ but that is not the same as the work done by F_2 acting on the left hand end of the spring.

The spring compresses by a distance $x_1 - x_2$, assuming that $x_1 > x_2$. At the end of the collision, the two balls separate when $x_1 = x_2$ and the forces F_1 and F_2 decrease to zero. While the spring is compressing, the work done by F_2 on the spring is $\int F_2\,\mathrm{d}(x_1 - x_2)$ and the work done by the spring on m_2 is $\int F_2\,\mathrm{d}x_2 = \frac{1}{2}m_2 v_2^2$ where $v_2 = \mathrm{d}x_2/\mathrm{d}t$ is the velocity of m_2.

Given that $F_2 = k(x_1 - x_2)$, where k is the stiffness of the spring, the elastic potential energy stored in the spring is equal to the work done on the spring, $W_1 = \int F_2\,\mathrm{d}(x_1 - x_2) = \frac{1}{2}k(x_1 - x_2)^2$ and the work done by F_2 to increase the kinetic energy of m_2 is $W_2 = \int F_2\,\mathrm{d}x_2$. The total work done by F_2 acting on the left end of the spring is therefore $W_1 + W_2 = \int F_2\,\mathrm{d}(x_1 - x_2) + \int F_2\mathrm{d}x_2 = \int F_2\,\mathrm{d}x_1$. That is, the total work done by F_2 at the left end of the spring is determined by the displacement of the point of application of F_2. The latter relation is rarely mentioned in physics textbooks but is often mentioned in physics journal articles as an alternative or additional work-energy theorem [2]. The usual work-energy theorem mentioned in textbooks refers only to the displacement of the centre of mass of an object. The textbook work-energy theorem indicates that the work done by F_2 at the right end of the spring, acting on m_2, is $\int F_2\,\mathrm{d}x_2$ where x_2 is the displacement of the centre of mass of m_2.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Received 16 November 2025
Accepted for publication 7 January 2026
https://doi.org/10.1088/1361-6552/ae3513

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