

Rolling friction on a wheeled laboratory cart

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Abstract

A simple model is developed that predicts the coefficient of rolling friction for an undriven laboratory cart on a track that is approximately independent of the mass loaded onto the cart and of the angle of inclination of the track. The model includes both deformation of the wheels/track and frictional torque at the axles/bearings. The concept of rolling friction is contrasted with the static or kinetic friction that in general is also present, such as for a cylinder or ball rolling along a horizontal or inclined surface.

Introduction

Small rolling carts on metal tracks have become common in introductory physics laboratories at universities and secondary schools. Examples include PASCO's 'Collision Cart', Vernier's 'Standard Cart', PHYWE's 'Low Friction Cart', and PSSC's 'Dynamics Cart'. Nevertheless, there exists confusion about how to simply and accurately model the resistance that such carts encounter when they move on level or inclined tracks.

For example, two articles [1, 2] refer to the friction experienced by the carts as 'kinetic', which is incorrect because they roll not slide along the track. A more recent paper [3] does not specifically use the adjective 'kinetic', but nevertheless subscripts the coefficient of friction between the cart and track with the letter ' k ' which presumably is an abbreviation for that adjective. Other authors hedge their bets by simply referring to the force opposing the motion of the carts as 'friction' without a qualifying adjective [4, 5] and the coefficient of friction by the unsubscripted symbol ' μ ' [6], even though most introductory physics textbooks only denote friction as either static or kinetic.

Part of the confusion is understandable, in that static or kinetic friction, under many circumstances, dominates and hence masks the rolling friction, which is always present and distinct from either of these other two kinds of friction for a rolling object. For example, if a string is wound around the axle of a rolling cylinder [7] or of a spool [8] and used to drive its motion, then static friction usually arises to prevent the object from slipping. (Depending on how it is pulled, static friction is not always needed [9], but usually it is.) However, static friction is not necessarily a 'resistance' to motion: in fact, it can point in (as opposed to against) the direction of motion [10]. In any case it does not dissipate any mechanical energy because its point of contact at the bottom of the rolling object has zero instantaneous velocity. (Zero displacement means zero work done. It is static after all.) Similarly, a cylinder rolling up or down an incline is driven by gravity, and a vehicle (such as a car or bicycle) accelerating along a level road is driven by a motor or by pedalling. In such cases, static friction again dominates (assuming the rolling is without slipping) and can point either forward or backward relative to the direction of motion [8]. If slipping does occur, then kinetic

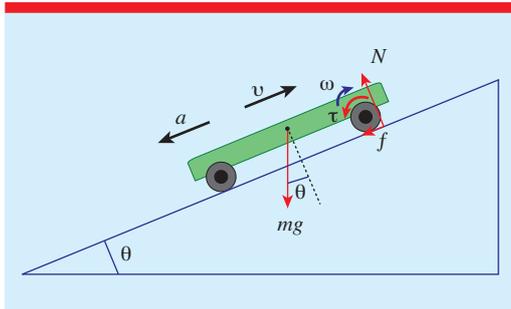


Figure 1. Free-body diagram for a cart rolling up an incline (so that the velocity v of the centre of mass of the cart points uphill, but since the cart is slowing down, its acceleration a points downhill). The normal force N , frictional force f and axle torque τ are shown at one of the front wheels (whose angular velocity is ω) but are actually sums over all four wheels.

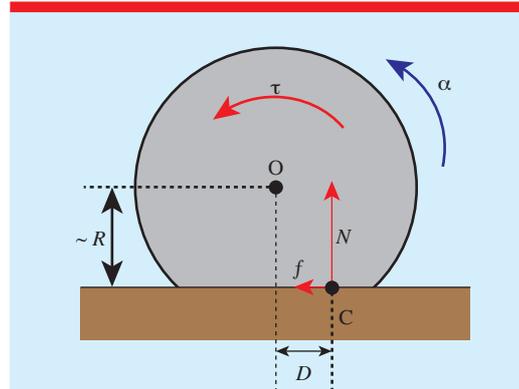


Figure 2. Enlarged view of the contact forces and axle torque at the wheels. This diagram has been rotated clockwise by angle θ compared to figure 1. The axle has been labelled as the origin O and the effective point of contact of the forces between the wheel and track as C, separated horizontally from each other by a distance D . The flattening of the wheel has been exaggerated for clarity. The direction of α is consistent with the direction of a in figure 1.

friction will dominate, as is purposely exploited in some games such as bowling and billiards [11].

In contrast, if a cylinder or ball is freely rolled (i.e. without being driven by a string or motor) along a horizontal surface, it will eventually come to a stop (even if it does not strike any small bits of dirt or other obstacles). This slowing will arise even in the absence of air drag, as one can verify by rolling the object inside an evacuated bell jar. In the absence of slipping, the resistive force cannot be ordinary static friction, as explained in figure 1 of [12]; for example, if the friction force pointed backward (to translationally decelerate the object), then it would simultaneously rotationally accelerate the cylinder about its centre, which cannot be right! The object could partially stick to the surface and have to be ‘peeled’ away from it and thereby get slowed down [13], but that cannot be a complete explanation, as nonadhesive rolling objects also slow down. A simple but comprehensive model of rolling friction is developed in this article.

General theory of rolling friction

Rolling friction on the wheels of a cart can be modelled using the ideas of Krasner [12]. There are two contributions to the friction: deformation of the wheels and/or track, and the torque at the axles (or bearings) opposing the rotation of the wheels. The deformation will be represented here as a flattening of the wheels, although in general it can also arise from the cart creating a slight

depression in (or bowing of) the track out of which it has to continuously roll.

Consider a cart of mass m that is free rolling up an incline, as sketched in figure 1. The total frictional force f on the cart from the track is assumed to oppose the direction of motion v of the cart. Similarly, the total frictional torque τ at the four bearings opposes the direction of rotation ω of the wheels. Force components perpendicular to the incline must balance so that

$$N = mg \cos \theta \quad (1)$$

where N is the sum of the normal forces at all four wheels. (In general, N , f and τ will not be equally divided among the four wheels.) Force components along the incline imply

$$mg \sin \theta + f = ma. \quad (2)$$

The rotational form of Newton’s second law about point O in the enlarged view in figure 2 is

$$ND + \tau - fR = I\alpha \quad (3)$$

where the angular acceleration of the wheels is $\alpha = a/R$ assuming they do not slip, and the sum of the moments of inertia of the four wheels is $I = \gamma MR^2$. Here M is the total mass of the four wheels (each of radius R) and γ is

a mass distribution factor, of the order of 0.5 if the wheels are approximately uniform discs. Solving equation (2) for a and substituting it into equation (3) leads to

$$ND + \tau - fR = \gamma MR(g \sin \theta + f/m). \quad (4)$$

Now suppose we can relate the frictional force on the cart to the normal force on the entire cart via a coefficient of friction μ (labelled without subscripts for the moment),

$$f = \mu N = \mu mg \cos \theta \quad (5)$$

using equation (1) to get the second equality. Similarly suppose that the frictional torque on the axles can be related to the normal component of the gravitational load on them from the body of the cart without the wheels, $(m - M)g \cos \theta$, so that

$$\tau = kr(m - M)g \cos \theta \quad (6)$$

where r is the radius of the axles and k is a (dimensionless) coefficient of frictional torque. It is assumed that μ and k are independent of speed. In support of that assumption, measurements of a ball on an incline [14] indicate that μ has less than a 10% dependence on speed for angular velocities up to 1.5 rad s^{-1} . Given that the wheels of a PASCO cart have a diameter of $2R = 2.54 \pm 0.01 \text{ cm}$, this limiting angular velocity requires that the cart travels slower than 2 m s^{-1} , which is true in typical introductory lab experiments. (For example, in figure 2 of [6], the cart speed never exceeds 0.6 m s^{-1} .) Evidence that k is independent of speed comes from measurements of the frictional torque on a physical pendulum [15].

Substituting equations (1), (5) and (6) into (4) gives rise to

$$\mu = \frac{\frac{D}{R} + k\frac{r}{R}(1 - \frac{M}{m}) - \gamma\frac{M}{m}\tan\theta}{1 + \gamma\frac{M}{m}} \quad (7)$$

after a little algebra. Equation (7) is a universal formula describing free rolling up an incline in the absence of air resistance. (One can verify that air drag is negligible compared to rolling friction at the speeds of motion of typical lab carts by substituting appropriate values into the formula given in example 6.8 of [16].) As a check, this formula correctly reduces to two previously published expressions as follows.

(1) Consider a single round object (so that $M = m$) rolling on a horizontal surface (so that $\theta = 0$). Notice that the term proportional to k in equation (7) then drops out, as expected since there is no axle. One finds that $\mu = (D/R)/(1 + \gamma)$, which is the coefficient of rolling friction (usually written as μ_r) that explains why the object will eventually roll to a stop [17]. This coefficient agrees with Krasner's equation (5) after correcting the typo whereby he multiplied (D/R) and $(1 + \gamma)$ instead of dividing them [12].

(2) Again consider a single round object (so that $M = m$) but this time rolling on an incline. Suppose the object and track are sufficiently rigid that they cannot deform (so that $D = 0$). Now equation (7) simplifies to $\mu = -\tan\theta/(1 + \gamma^{-1})$. The minus sign implies that the frictional force is directed up rather than down the incline. This upward direction is necessary so that friction will angularly decelerate the object as it rolls up the ramp; the friction in this case is static not rolling [8]. We see that μ is not a constant independent of θ , but instead its value automatically adjusts (up to a maximum value of μ_s if slipping is not to occur). It is more conventional to write $f_s = \mu N = -mg \sin\theta/(1 + \gamma^{-1})$ from equation (5) and not refer directly to μ at all. Substituting this expression for f_s into equation (2) leads to the familiar result $a = g \sin\theta/(1 + \gamma)$ for the translational acceleration of a rolling object on an inclined plane.

As a specific example, consider a standard PASCO Collision Cart (model number ME-9454). Then equation (7) can be numerically approximated as follows. The wheels are hard plastic discs so that $\gamma \approx \frac{1}{2}$. I dismantled a cart and measured the plastic part of a wheel to have a mass of $1.74 \pm 0.02 \text{ g}$ and a bearing to have a mass of $0.72 \pm 0.01 \text{ g}$. Adding these together and multiplying by 4 gives the mass of all the rotating parts as $M = 9.84 \pm 0.12 \text{ g}$. (Only a portion of each bearing rotates, so this value is actually an upper limit on M .) The total mass of a cart with its wheels is $m = 0.5015 \pm 0.0004 \text{ kg}$. Consequently $M/m \approx 0.02 \ll 1$ and equation (7) becomes

$$\mu \approx \frac{D}{R} + k\frac{r}{R} - \gamma\frac{M}{m}\tan\theta. \quad (8)$$

Experimentally it was found that $\mu = 0.0065 \pm 0.0002$ by rolling a cart along a level track and fitting a straight line to the speed squared as a

function of position, both measured using a motion detector [18]. (The same value of μ was found for a variety of different cart speeds, up to at least 0.7 m s^{-1} .) However, for values of θ up to 5° (which is about the largest track tilt one would realistically use for a free rolling cart), $\gamma(M/m) \tan \theta \leq 0.0009$ and thus the final term in equation (8) only makes a small contribution to the overall sum. Neglecting it results in

$$\mu_r \approx \frac{D + kr}{R} \quad (9)$$

where a subscript 'r' has been added to denote that this is now the coefficient of rolling friction. This final expression is independent of the angle of inclination, justifying the experimental method mentioned above in which the coefficient was measured on a horizontal track. It is also independent of the mass loading of a cart. Equation (9) is a simple, dimensionally correct sum of contributions due to contact deformation and axle friction.

It is left as an exercise for the reader (or her students) to redo the analysis presented here for the case of a cart free rolling down (rather than up) an incline. The directions of the arrows for v , ω , τ and f need to be reversed in figures 1 and 2, and point C needs to be moved to the downhill (rather than the uphill) side of the wheel, a distance D to the left of point O. Equation (7) then becomes

$$\mu = \frac{\frac{D}{R} + k \frac{r}{R} (1 - \frac{M}{m}) + \gamma \frac{M}{m} \tan \theta}{1 + \gamma \frac{M}{m}} \quad (10)$$

which differs only in the last sign in the numerator. (That difference can be rationalized simply as a change in the sign of θ to transform an uphill to a downhill inclination.) But that last term is dropped in deriving equation (9) and hence that final expression for μ_r is valid for both uphill and downhill motions of a cart.

Conclusions

A universal equation (7) has been found for the ratio of the effective frictional force to the normal force on an object freely rolling along a horizontal or inclined track. If the object is a standard lab cart, this ratio is equal to the coefficient of rolling friction μ_r for realistic angles of inclination. The fundamental reason that rolling friction dominates the static friction in this case is that the wheels

(which are the rotating components) make up only a few per cent of the total mass of the cart.

On the other hand, if the object is a single rotating object such as a cylinder or ball, then there can exist both a significant static frictional force of magnitude $f_s = mg \sin \theta / (1 + \gamma^{-1})$ and rolling frictional force $f_r = mg \cos \theta (D/R) / (1 + \gamma)$, according to equations (5) and (7). Their ratio is

$$\frac{f_s}{f_r} = \gamma \frac{R}{D} \tan \theta. \quad (11)$$

Normally D is much less than R (see figure 2) and the static friction dominates unless the track is horizontal or nearly so (i.e. $\theta \approx 0$). However, both are actually present, as illustrated in figure 10.20 of [19]. Rolling friction manifests itself as a loss of mechanical energy as a ball descends a ramp, because there is no such dissipation under the idealized conditions usually discussed in the classroom. In particular, if the familiar demonstration of a marble on a loop-the-loop track is performed [20], one quickly discovers that the marble has to be released from a height well above the idealized prediction of 2.7 times the radius of the loop. (Much closer agreement with idealized theory is obtained if the demonstration is instead performed using an interrupted pendulum [21], since air drag is much weaker than rolling friction.)

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