Pressure Change in an Arterial Constriction

Carl E. Mungan, U.S. Naval Academy, Annapolis, MD

Consider the following ConceptTest. A platelet is drifting with the blood flowing through a horizontal artery. As the platelet enters a constriction, does the blood pressure increase, decrease, or stay the same?

Figure 1 sketches the situation. The blood is assumed to be incompressible and to have the density \( \rho = 1000 \text{ kg/m}^3 \) of water. Also, for simplicity the pressure \( p \) is taken to be uniform across any constant-diameter region of the artery, by neglecting boundary layer drag at the surfaces of the artery, and the flow is assumed to be laminar and inviscid.

The correct answer comes from combining the hydrodynamic equations of continuity and of Bernoulli. Suppose that at the constriction the cross-sectional area \( A \) of the artery is reduced to a fraction \( f \) of its original value. Then the speed \( v \) of the blood must increase in the constriction by a factor of \( 1/f \) to maintain a constant volumetric flow rate. But Bernoulli’s equation says the sum \( p + \frac{1}{2} \rho v^2 \) is a constant along the horizontal streamline followed by the platelet. The increase in the second term in the sum must be balanced by a corresponding drop in the first term. That is, the pressure is lower in the constriction than in the rest of the artery. The drop in pressure as the platelet enters the constriction means there will be a larger force on the trailing face than on the leading face of the platelet, which is what accelerates it to a higher speed. Likewise, the rise in pressure slows the platelet back down as it leaves the constriction.

Having polled this question in an introductory calculus-based physics class for non-majors using clickers with a discussion of the preceding correct answer, I put it on a test two weeks later. For full credit, students had to explain their reasoning. Interestingly, 21 out of the 43 students in the course provided one of the following wrong but plausible answers. Careful consideration of the thinking behind these two answers could help improve instruction in introductory physics courses.

One incorrect answer provided by 12 students went as follows. Pressure equals force divided by area. The artery’s cross-sectional area is smaller in the constriction, and thus the pressure must be larger there. These students are trying to use the definition of pressure to relate the two key quantities of interest in the problem, \( p \) and \( A \). The flaw in reasoning is that the students have assumed the force \( F \) on a cross-sectional slice of fluid to be the same in both regions of the artery. In fact, both \( p \) and \( A \) are smaller in the constriction, and hence so is \( F = pA \), compared to values in the large-diameter portions of the artery. (The appendix works out the exact percentage by which the force drops in the constriction.) The students ignored a third quantity \( (F) \) in a relationship between two variables \( (p \) and \( A \)). They are familiar with functions having only one dependent and one independent variable, \( y \) and \( x \), from their math courses, with other symbols being constants. An example is the equation \( y = kx^2 \) describing a parabola; one can correctly conclude that if you double the horizontal distance \( x \) from the origin to a point on the parabola, the vertical distance \( y \) quadruples.

The preceding answer is based on incorrect proportional reasoning about a mathematical formula. The second wrong answer provided by nine students, in contrast, stems from erroneous conceptual thinking. These students are visualizing a number of platelets distributed in the blood. They reasoned that the platelets get squeezed closer together in the narrow constriction, and hence the fluid pressure must rise there. It is true that the fluid gets “squeezed” in the direction perpendicular to the blood flow. However, that is compensated by the fact that the fluid gets “stretched” in the longitudinal direction by its increased speed through the constriction. The second effect must exactly balance the volume change of the first effect, as otherwise the density of the fluid would change! In contrast, the students are thinking of the platelets as cars driving down a four-lane highway and approaching a construction zone in which only two lanes are open so that the cars bunch together as they pass through it. But on a real highway, the automobiles would have to slow down even before reaching the danger zone. Evidently cars do not satisfy the equation of continuity in the form \( Av = \text{constant} \), because the density of cars varies. The familiar analogy drawn between an incompressible fluid and automobile traffic only holds if drivers always maintain a fixed average distance away from the nearest cars (or the edge of the highway if they are in the outermost two lanes) in front of, behind, and on both sides of themselves. Now suppose an accident occurs that suddenly blocks some lanes on the highway. A pressure wave will propagate backward from this constriction and slow the incoming traffic. The cars cannot maintain a fixed speed and density both before and through the constriction without plowing into the blocked lanes! In the fluid case, the normal force exerted backward on the liquid by the necked-in walls of the pipe at the beginning of the constriction in Fig. 1 adds...
to the pressure on the inlet side (relative to that inside the constriction). Likewise, there is an additional forward normal force due to the necked-out walls on the outlet side at the end of the constriction that increases the fluid pressure after it.

Appendix

Let subscript “L” refer to the large-diameter regions of the artery, and subscript “S” to the small-diameter regions, as in Fig. 1. The equation of continuity is

\[ A_L v_L = A_S v_S \Rightarrow v_S = v_L / f, \]  

(1)

where \( f = A_S / A_L \). For a horizontal pipe, Bernoulli’s equation is

\[ p_L + \frac{1}{2} \rho v_L^2 = p_S + \frac{1}{2} \rho v_S^2. \]  

(2)

Substitute Eq. (1) into the last term, replace each pressure by force divided by area, and define \( \gamma \equiv \rho v_L^2 / 2p_L \) to end up with a fractional force reduction of

\[ \frac{F_L - F_S}{F_L} = \left(1 - f\right) \left(1 + \gamma + \frac{\gamma}{f}\right). \]  

(3)

This solution is necessarily positive because \( 0 < f < 1 \) and \( \gamma > 0 \). For example, the blood in the unconstricted portion of the aorta has a mean gauge pressure\(^6\) of 93 mm Hg so that the absolute pressure is \( p_L = 113 \text{ kPa} \) at a speed\(^7\) of \( v_L = 11 \text{ cm/s} \). Suppose that deposits on the arterial walls reduce the cross-sectional area to a fraction \( f = 1/3 \). For these values, \( \gamma \approx 5 \times 10^{-5} \ll (1 + 1/f)^{-1} \) and hence Eq. (3) simplifies to \( F_S / F_L \approx f \), corresponding to a force reduction within the constriction of 67%.

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References

2. Many standard physiology textbooks incorrectly make the unqualified statement that blood flows from regions of higher to lower pressure in the body, in contrast to what happens as the blood leaves the constriction in Fig. 1. For example, see G.J. Tortora and S.R. Grabowski, Introduction to the Human Body: The Essentials of Anatomy and Physiology, 5th ed. (Wiley, New York, 2001), p. 376.
7. http://circ.ahajournals.org/content/40/5/603.