Sampling discretization can lead to strange shapes of data curves

The January 2016 “Little Gems” column\(^1\) employed a Vernier light sensor to measure the variation in the illuminance of a strobe light at a low, medium, and high rate of flashing. A strange alternating narrow-single and broad-double peaked structure was observed at the high rate of flashing, as reproduced in the top panel of Fig. 1 shown here. This graph was mocked up by assuming the illuminance is zero when the lamp is off and 12 kilolux when it is on, and that the red data points are collected at a sampling rate of 20 points/s over the range from 0 to 2 s. LoggerPro defaults to connecting adjacent data points by straight line segments, resulting in the anomalous blue curve. However, the bottom panel of Fig. 1 demonstrates that these red data points are consistent with a 6-Hz square wave. Motivated by this idea, we returned to the apparatus and verified that the sampling rate was indeed 20 points/s. Increasing it to 1000 points/s (which the Vernier light sensor is capable of handling) gives the predicted square wave.


**Fig. 1.** The top graph shows the same data as in Fig. 2(c) of Ref. 1 except that the data points have been highlighted in red; the blue line segments connect adjacent data points. The bottom graph presents the same red data points but now with a 6-Hz square wave drawn through them with its first downward transition at 0.03 s. Both graphs repeat their overall pattern of red dots and blue lines every \( T = 0.5 \) s because the smallest positive integers that satisfy the commensuration relation \( T = n/(6 \text{ Hz}) = m/(20 \text{ Hz}) \) are \( n = 3 \) and \( m = 10 \). (Here 6 Hz is the frequency of the square wave and 20 Hz is the sampling frequency of the data points.)

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